Measuring DC Resistance of Calibration Loads

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Introduction

For calibration of a VNA with OSL (aka SOL) calibration, we need Open, Short and Load calibration standards. Ideally, the Load has exactly 50 ohms resistance over the full frequency range. However, it is common to use commercial terminations as calibration Loads. They may have DC resistance in the 48-52 ohm range (or even a broader range). The more precisely we can specify the resistance to the VNA, the more accurate our measurements will be. It may also be necessary to specify other high frequency characteristics as well, such as a bit of capacitance. But the starting point is to get the DC resistance correct.

Our goal here is to present simple and inexpensive methods to measure Load resistance, hopefully with accuracy of 0.05%-0.1%. If we specify the Load resistance with error of only 0.05%, we should get comparable results to using a near-perfect Load that is actually within 0.05% of 50 ohms.

We will take advantage of the fact that our range of interest is fairly narrow; we limit it to the range of approximately 48-52 ohms, which will include most terminations that we would consider using as calibration Loads. But our circuit is also easily modified to cover Loads near 75 ohms.

Meters

The most common style of meter is probably the 3 ½ digit digital multimeter (DMM). A 3 ½ digit DMM will display from 0 to 1999, with the decimal point depending on the range. In the 200 mV DC voltage range, this gives 0 to 199.9 mV. A typical accuracy spec for such a meter would be 1% plus 2 digits. The 1% is normally proportional to the measurement being made, (unless it is expressly based on Full Scale). This is essentially uncertainty about the scaling of the reading, caused by things such as the tolerance of internal resistors; we refer to this error as the “Scaling Error”. The “plus 2 digits” indicates that whatever the reading, there is additional possible error of 2 least-significant digits. In the above 200 mV range, this is an error of +/-0.2 mV. We refer to this error as the “Digit Error”, and it is likely due primarily to non-linearities of the digital conversion.

A DMM will also resistance, but the accuracy specs are generally much weaker for these measurements. A 200 ohm range on a 3 ½ digit meter can display to one-tenth of an ohm, but the Digit Error may be 5 or more digits, representing half an ohm.

A 4 ½ digit voltmeter will display from 0 to 19999, with the decimal point depending on the range. In the 200 mV range, this gives 0 to 0.19999 volts, or 199.99 mV. The 200 ohm resistance range will read to 199.99 ohms. Because the Digit Error in a 4 ½ digit meter affects a digit with such a small value, it is frequently of little concern. In addition, the Scaling Error of such meters is usually much less than that of the lower resolution meters. On the other hand, there are cheap 4 ½ digit meters that are essentially 3 ½ digit meters that display an extra, meaningless decimal place.

Our measurement task is obviously much easier with a good higher-resolution meter. The primary goal of this document is to show how to get good measurements out of a 3 ½ digit meter, resulting in final accuracy that is far better than the basic accuracy of the meter.

I use two DMMs. One is an old, modestly priced off-brand “3 ½ digit. The other is a B&K 391A, a 4 ½ digit meter costing $195.

Straightforward Resistance Measurement

We should first consider directly measuring the resistance. The specs for most reasonably priced meters are nowhere what we need for resistance measurement. However, when measuring resistances near 50 ohms, the actual accuracy of both my meters is very good, with no apparent Scaling Error. The 3 ½ digit meter will read 50.1 for a 50 ohm resistor, and the 4 ½ digit meter will read 50.02 ohms. Knowing the amount of error, it might be feasible to directly measure resistance. When using the low-resolution meter I could subtract 0.1 ohms from the measurement; when using the high-resolution meter I could subtract 0.02 ohms.

This assumes that over a narrow range of resistance, the error factor remains almost constant. This is likely to be the case for analog error, but analog-to-digital converters can have abrupt changes, such as might occur when moving from two nearby values whose binary bit configurations are significantly different. Still, while such digital errors can be abrupt, they typically would be no bigger than a few binary digits. In fact, it is likely that they would be no larger than the Digit Error of the DMM. For the 4 ½ digit meter, which reads to hundredths of an ohm, this error is quite small. For a 3 ½ digit meter, reading to tenths of an ohm, this error can be several tenths of an ohm, perhaps amounting to 0.5% of the reading.

Great theory, but it is a good to verify the basic operation of the meter. I constructed the resistor ladder in Appendix A, which exposes resistances from 46 to 55 ohms in very precise one-ohm steps. The accuracy is very near 0.02%, and all the resistors cost a total of about $10.00. At all resistances, my 4 ½ digit meter maintained constant error of 0.02 ohms; that 0.04% error could largely be removed by subtracting 0.02 ohms from each reading. The 3 ½ digit meter varied back and forth between 0.1 and 0.2 ohms, so if I subtracted 0.1 ohms from the readings, I would still have an occasional 0.1 ohm error (0.2%).

Conclusion: Direct resistance measurement, with the above described error correction, will give good results with this particular 4 ½ digit meter. It will also improve the accuracy of the 3 ½ digit meter, though not to the 0.1% level. Had the test with the resistor ladder shown that the error of a meter varied back and forth significantly as we stepped through the resistances, this method would not be suitable.

Caveat: Use the shortest leads possible. One of my DMMs has crazy-long 4' leads, which is ridiculous for precise resistance measurements. The resistance in the leads adds to the resistance being measured. Note that when doing voltage measurements, lead resistance is trivial compared to the typical 10 Mohm input impedance of the meter. Therefore, for the remainder of this document, which deals with voltage measurements, lead length is not a concern.

Voltage Measurement Method

Our next method relies on creating a basic voltage divider with the Load (R_{Load}) as the grounded leg, measuring a couple of voltages and then calculating R_{Load}. A 4 ½ digit meter can possibly measure the voltages directly with good results, but we need a measurement trick to improve measurements from 3 ½ volt meters.

Figure 1 shows the schematic of our measurement circuit, which we discuss below.

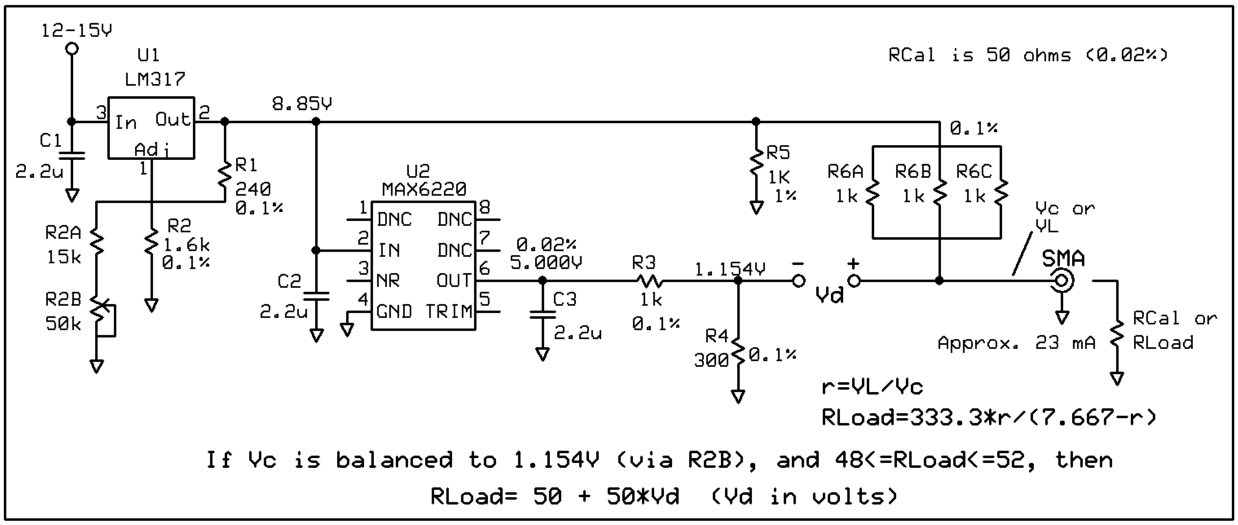


Figure 1—Schematic of Measurement Circuit

A good choice for R_{Cal} is two parallel Susumu RG2012V-101-P-T1 (100 ohms)

The circuit uses an LM317 to get a stable but slightly adjustable 9V supply, which feeds the precision voltage reference (U2). The 9V supply also feeds a voltage divider that is 268 ohms on the top (R6A|| R6B||R6C), and on the bottom has either a 50-ohm calibration resistor or our target R_{Load}. The voltage across the calibration resistor will be adjusted to 1.154. The voltage divider consisting of R3 and R4 will also produce a reference voltage of 1.154 volts. We can measure the voltage across the calibration resistor (V_C) or the Load (V_L) by measuring Vd and adding 1.154 to the result. Any meter error in the measurement will be based on the relatively small size of Vd (max of about 50 mV), and will thus be a small percentage of the total voltage.

Before measuring R_{Load}, we attach the calibration resistor and adjust R2B so that Vd=0, which means V_{Cal}=1.154 volts. We then replace the calibration resistor with R_{Load} and measure Vd again. We can add 1.154 volts to that reading to get V_L, and then apply the formulas in Figure 1 (see Appendix B for derivation) to calculate r and then R_{Load}:

$$r = \frac{V_L}{V_C} \qquad R_{Load} = \frac{333.3 \cdot r}{7.667 - r}$$

Eq. 1--Calculating near 50-ohm resistances from a ratio of voltages

This equation is accurate for any R_{Load} value, but component tolerances and meter error limit us to measuring Loads within a few ohms of 50. There is a more direct route, which calculates R_{Load} directly from Vd, as shown in the bottom of Figure 1. That formula is quite accurate for R_{Load} between 48 and 52 ohms, which is our area of concern. That formula is:

$$R_{Load} = 50 + 50 \cdot Vd \quad (Vd \text{ in volts; } 48 \le R_{Load} \le 52)$$

Eq. 2--Calculating near 50-ohm resistances directly from Vd

Eq. 2 turns a non-linear relationship into a linear equation, but has mathematical error no greater than 0.03%; the worst errors occur furthest from 50 ohms. In addition to this mathematical error, we have certain measurement errors. Errors in the 1.154 voltage reference largely cancel out because they have the same proportional impact on V_C and V_L. Similarly, errors in the values of R6A and R6B have very tiny impacts on the results, again because they tend to impact V_C and V_L nearly equally. In both cases, these errors have minimal impact primarily because our range of resistances is small.

The primary remaining errors are the Digit Error and Scaling Error when measuring Vd to determine V_L. We can measure with the 200.0 mV range of the meter, so if there are a few digits of error, that will be only a few tenths of a mV. That error gets multiplied by 50 in the calculation, so 2 mV of error is 50*0.0002 or 0.01 ohms, which is negligible. The Scaling Error can be more significant, if Vd is on the large end of its range. At 0.05 V, with a 1% meter, the Scaling Error is 0.0005V, which when multiplied by 50 is 0.025 ohms, or 0.05% of 50 ohms. So near the ends of the resistance range the total possible error can approach 0.1%, but closer to 50 ohms it may be limited to 0.06%.

Some test results of this circuit are shown in Appendix C.

Modifications for 75 ohms

We have been targeting Loads near 50 ohms, but we may also be interested in those near 75 ohms. To measure those, all we have to do is remove R6C from the circuit, use a 75 ohm calibration resistor, and modify our formulas:

$$r = \frac{V_L}{V_C} \qquad R_{Load} = \frac{500 \cdot r}{11 - r}$$

Eq. 3--Calculating near 75-ohm resistances from a ratio of voltages

The formula for calculating R_{Load} directly from Vd also changes:

$$R_{Load} = 75 + 75 \cdot Vd \quad (Vd \text{ in volts; } 73 \le R_{Load} \le 77)$$

Eq. 4--Calculating near 75-ohm resistances directly from Vd

Absolute errors will be somewhat larger when measuring near 75 ohms because any error in Vd gets multiplied by 75 rather than 50. However, as a percent of the total resistance, the error will be nearly the same as for 50 ohm measurements.

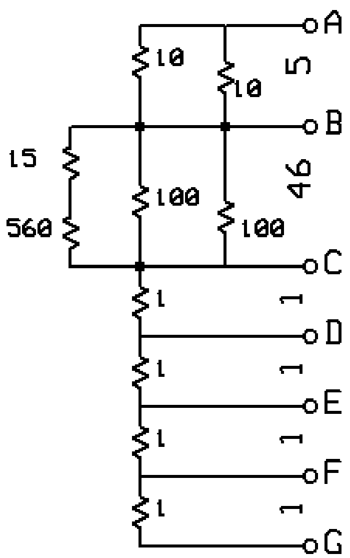
Conclusion

We have shown that it is possible to get accurate resistance measurements (better than 0.1%) of calibration Loads with a 4 ½ digit meter. Those measurements can exceed the accuracy specs of the meter, by largely "calibrating out" the error. The same technique can be used with 3 ½ digit meters, but their limited resolution makes it difficult to get better than 0.3% accuracy when measuring 50 ohms.

We can improve accuracy of 3 ½ digit meters by measuring voltage instead of resistance, and measuring that voltage against a precision reference rather than against ground. That reduces the magnitude of the voltage measured by the meter, reducing the error in the measurement so that it is a relatively small percentage of the total voltage being measured. This can improve the accuracy of resistance measurements to better than 0.1%, even with a meter whose basic accuracy is much worse than that.

Our technique relies on the fact that we are normally targeting a small range of resistances around 50 ohms, but it only requires removal of one resistor to re-target the circuit to the area around 75 ohms.

APPENDIX A—Resistor Ladder For 46-55 ohms



Most cost-effective components at this time;
prices and part numbers are Mouser:

1 Ohm (0.5%, $0.79):
Vishay 71-CPF1-D-1.0-E3 (Through hole)

10 Ohms (0.1%, $0.48):
Vishay 71-CPF1-B-10-E3 (Through hole)

15 Ohms (0.1%, $0.93)
IRC 66-PFC12C15-B (SMT 1206)

100 Ohms (0.02%, $3.15):
Susumu 754-RG2012V-101-P-T1 (SMT 0805)

560 Ohms (0.1%, $0.64):
Vishay 71-TNPW080560BEEA (SMT 0805)

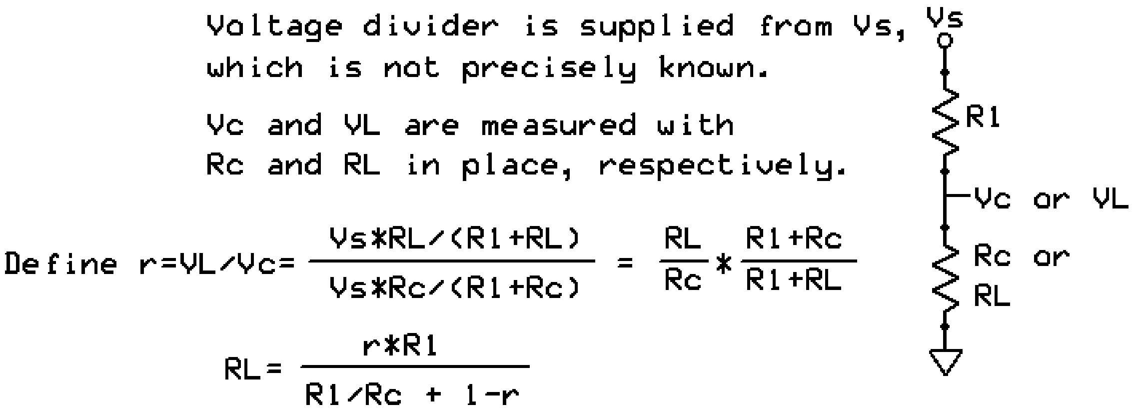
For resistances 46-55 ohms,
measure between these terminals.

46	B,C	51	A,C
47	B,D	52	A,D
48	B,E	53	A,E
49	B,F	54	A,F
50	B,G	55	A,G

Gold shorting links (Mouser 855-D3082-05) soldered at the resistance nodes make a good place for attaching the DMM leads with alligator clips.

APPENDIX B—VOLTAGE DIVIDER FORMULA

Here is the derivation of the formula used to calculate R_{Load} in Figure 1:



For us, R_C is always 50 ohms, and r is always within 0.04 of 1. The precise value of R1 is not critical, as it is the dominant term in both the numerator and denominator, so any deviations from ideal tend to cancel out. For example, if R_C=50 and r=1.04, the equation is

$$R_L = \frac{1.04 * R_1}{R_1 / 50 - 0.04}$$

An R1 value of 300 produces RL=52.35 ; 310 produces RL= 52.34. If we use this equation assuming R1=300, when it is in fact 310 ohms, this 3% error in R1 produces only a 0.02% error in our calculated R_L. If R1 has 0.1% tolerance, it contributes virtually nothing to the possible error.

APPENDIX C—TEST RESULTS

I used two methods to determine the resistance of a number of “50-ohm” terminations, and got the following results:

Vd (mV)	R Formula	R Meas	Error
-93.36	46.19	46.17	0.03%
-4.20	49.83	49.84	-0.02%
-2.15	49.91	49.89	0.04%
-0.50	49.98	49.97	0.02%
0.00	50.00	50.00	0.00%
2.20	50.09	50.06	0.06%
6.00	50.25	50.22	0.05%
8.60	50.35	50.34	0.03%
100.30	54.20	54.19	0.02%

Table C-1—Test Results

One method was direct measurement with a 4 ½ digit meter, calibrating it against a 50 ohm, 0.01% resistor that showed the meter read 0.01 ohms high (which value was then subtracted from measurements). Those values are listed as “R Meas”. The other method was that of Figure 1, measuring the voltage Vd (again with the 4 ½ digit meter). The resulting Vd values are also shown in the table, as is the R value calculated from Vd by formula. The formula is the simplified Eq. 2, except for the two extreme values; the fancier Eq.1 was used for those because they are too far from 50 ohms for the simplified approximation.

(These tests were done with an earlier prototype with slightly different component values and slightly different coefficients in Eq. 1.)

The final column shows the percentage error, which shows close agreement between the two methods. To the extent there is error, I suspect it is mostly due to Digit Error in the direct measurement, since the inherent meter accuracy is much better with voltages than resistances. It was particularly surprising to find the errors for resistances barely larger than 50 ohms. Nevertheless, the total error is small.

I didn’t do tests with a 3 ½ digit meter, but I did add some additional error to Vd—1% of the Vd reading plus an additional 0.5 mV (simulating 5 digits of error)—and reran the above table. All the errors remained under 0.1% except the very last entry, which increased to 0.14%. The resistance of 54 ohms is twice the distance from 50 that we were targeting, so a 1% error in measuring that deviation is twice what we allowed for. Still, 0.14% is decent accuracy.

I also tested the stability of the circuit of Figure 1. From a cold start, it requires about 2 minutes to stabilize, mostly due to the LM317. During that time, Vd (with the 50-ohm cal load) changed by 0.7 mV, and half that change occurred within the first 30 seconds. It is a good idea always to have a load attached to maintain thermal equilibrium, though when I removed one load and attached another there didn’t seem to be any need to wait for stabilization. (Other than the fact that 4 ½ digit meters can take a few seconds to reach their final reading.)

Adjustment of the variable resistor to get Vd=0 with a 50 ohm calibration resistor attached, is somewhat touchy, but would be much less so with a 3 ½ digit meter, which would not show deviations under 0.1 mV. There is also some drift that might require readjustment, but even drift of 0.2 mV has minimal impact on measurement accuracy. Many measurements can be made before that amount of drift accumulates, but it is a good idea to re-measure the calibration resistor just to be sure drift has not become excessive.