

## Measurement of Inductor Q with the MSA

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The Q of an inductor, which is its reactance divided by its internal series resistance, is used as an indication of how well it will perform at or near the resonant frequency when it is combined with a capacitor. This is relevant to LC filters and impedance matching circuits. Typically, we assume that the Q of the resonant circuit will equal that of the inductor, since capacitors typically have far higher Q than inductors and don't significantly degrade the circuit performance. However, to be precise, the  $Q_{res}$  of the resonant circuit will be as follows:

$$Q_{res} = \frac{Q_{Ind} * Q_{Cap}}{Q_{Ind} + Q_{Cap}}$$

### Equation 1—Determining Resonant Q from Inductor Q and Capacitor Q

$Q_{Ind}$  and  $Q_{Cap}$  are the Q values for the inductor and capacitor.

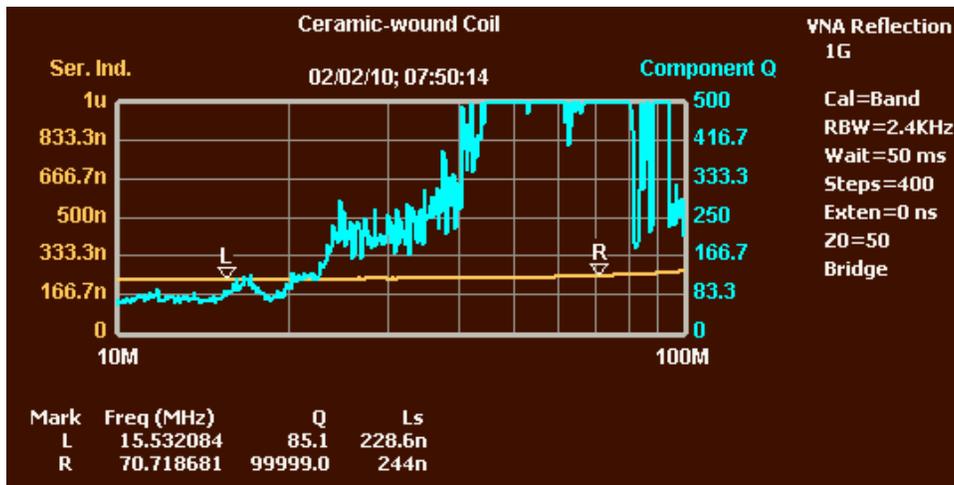
This document describes several methods that can be used to measure inductor Q using the MSA.

#### Direct Measurement in Reflection Mode

The simplest way to measure inductor Q is to directly measure the  $S_{11}$  of the inductor in Reflection mode, and graph the equivalent series resistance and reactance. Q is then the reactance divided by the resistance. The MSA will construct those graphs, and will also do the math and construct the Q graph.

This approach will work fine within a limited frequency range (well below the self-resonant frequency), with inductors whose Q value is not extremely high. It is difficult to get an accurate measurement of resistance when the resistance is completely swamped by the reactance; for high-Q inductors, the reactance is by definition far higher than the resistance.

Figure 1 shows such a measurement for a hand-wound coil with #14 wire on a ceramic form. The series inductance and the inductor Q are graphed:



**Figure 1—Inductance and Q of a Hand-Wound Inductor.**

In Figure 1, Q is labeled “Component Q”. It is probably accurate up to values of 40, and reasonably accurate where it measures up to 85, but above that the determination of the resistive component becomes more questionable, and the higher Q values become erratic. This sort of measurement is adequate for modestly-high Q values, but not for pinning down very high Q values. Still, even though we cannot be confident that the graph accurately shows Q at 80 MHz, we can conclude that Q is higher than 100, which in many cases is all we need to know.

The Q graph of Figure 1 is consistent with an inductor whose losses do not increase significantly with increasing frequency, so the increase in reactance causes a corresponding increase in Q (at least to 40 MHz—it is hard to say exactly what is happening above that).

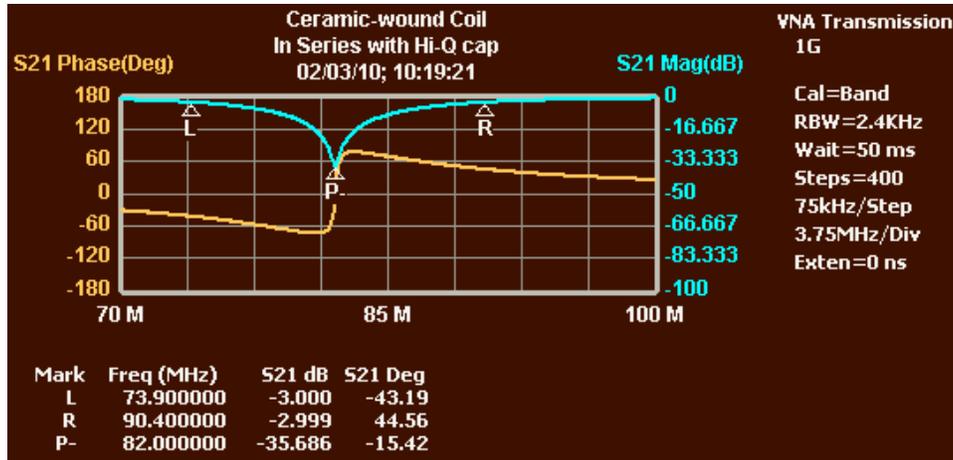
Note that the measured L value is increasing near the right side of the graph. This is likely due to the effect of the parallel parasitic capacitance of the inductor, which creates a self-resonant frequency (SRF) and causes the reactance to increase as we approach the SRF. The measurement of L can be affected even at frequencies as low as SRF/10. As discussed at the end of this document, accurate measurement when the apparent L value is rapidly rising requires use of RLC Analysis with a capacitor in parallel with the inductor.

### Transmission Mode RLC Analysis

We could get a better measure of the series resistance if it were not swamped by the series reactance. One way to get rid of the reactance is to add a series capacitor that will cause the circuit to resonate at or near the frequency of interest. (We want to use a capacitor with very high Q, such as the Cornell-Dubilier MC series of mica capacitors.) If we knew the L value, we could then measure the series resistance, calculate the inductor reactance, and divide reactance by resistance to get Q. But it is more useful to also measure the L value itself. We can’t simply use the above method to measure L and R at resonance, because the reactance at resonance will be zero, giving no information about the size of the inductance.

We can measure the value of all the components in Transmission mode by scanning the area around resonance (after first calibrating in the normal manner with the Through) and then using Functions→RLC Analysis. This will calculate the R, L, C and Q values for us.

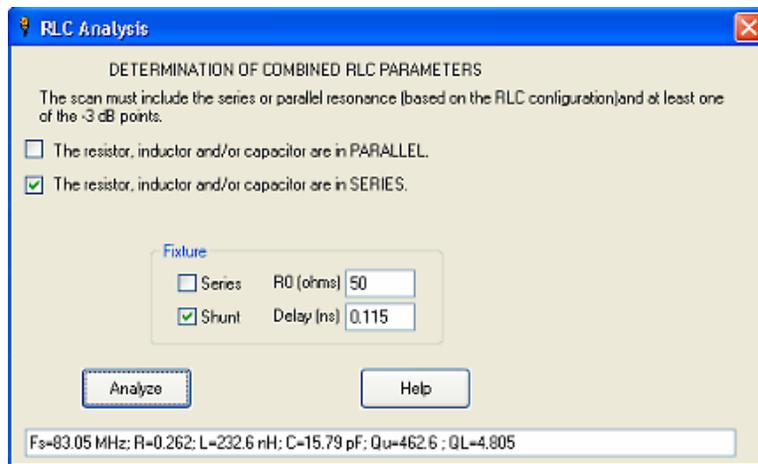
Figure 2 shows a Transmission mode scan of our inductor with a series capacitor added to create resonance near 80 MHz (the precise capacitor value was not known, but ends up being determined by RLC Analysis):



**Figure 2— $S_{21}$  of Inductor+15 pF Series Capacitor in Shunt Fixture**

Figure 2 was created by putting the series inductor+capacitor in a shunt fixture, which is similar to connecting the Tx and Rx lines of the VNA to one end and grounding the other. However, in the series fixture, attenuators are included in both the Tx and Rx lines to be sure the device being tested sees 50 ohms looking toward either Tx or Rx. The markers in Figure 2 were actually placed automatically by the RLC Analysis we are about to describe.

After creating the  $S_{21}$  scan, we invoke Functions→RLC Analysis. In the MSA version that created Figure 2, the following dialog was shown:



**Figure 3—Transmission Mode RLC Analysis Dialog  
(The current dialog is somewhat different.)**

The dialog allows us to specify whether the circuit is to be analyzed as a parallel or series RLC circuit and whether it is in a series or shunt fixture. We can also specify the  $R_0$  of the fixture, which is the resistance the RLC device sees looking toward the Tx or Rx ports (through the attenuators.) Finally, for the shunt fixture we can specify the one-way connector delay caused by

the connector that attaches our device to the fixture. 0.115 ns is typical for a fixture that has a female SMA connector, with the RLC device attached to the back of a mating connector. When we click Analyze, the results of the analysis are shown in the text box.

The first step of the analysis is for the software to locate the resonant frequency and -3 dB points, which are marked in Figure 2. Note that the -3 dB points for a notch response are at the absolute -3 dB level, not 3 dB above the bottom of the notch. The current software lets you choose which 3 dB points to use, and applies the appropriate formula. It is actually better to use the points that are 3 dB above the bottom of the notch, as long as the notch is at least 20 dB deep. One reason is that it allows the scan to concentrate on a smaller area around the notch, so relevant points can be more precisely located.

The results show R=0.26, L=233 nH, C=15.8 pF and  $Q_U=463$  at 83 MHz. This lends credibility to the sky-high Q values shown near 83 MHz in Figure 1.

Note that the  $Q_U$  produced by RLC Analysis is that of the inductor/capacitor combination. If the capacitor has an extremely high Q, the  $Q_U$  of the combination will pretty much equal the inductor Q. But with a  $Q_U$  of 463, it is very difficult to have a capacitor whose Q has a negligible effect on the circuit. If we have an idea of the capacitor Q, we can make the following adjustment to get inductor  $Q_{Ind}$ .

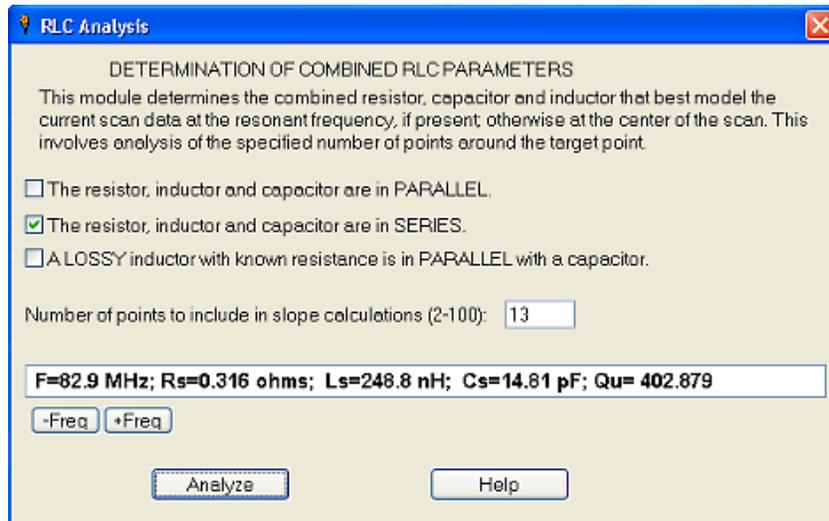
$$Q_{Ind} = \frac{Q_U Q_{Cap}}{Q_{Cap} - Q_U}$$

**Equation 2—Determining Inductor Q from  $Q_U$  and Capacitor Q**

Applying this adjustment will result in the inductor Q being even higher than the measured  $Q_U$  of the LC combination.

**Reflection Mode RLC Analysis**

We can also perform RLC Analysis in Reflection mode, and there are some advantages to doing so. The OSL calibration in Reflection mode removes many possible sources of error, such as the connector delay and the non-50-ohm source and load impedances of the fixture. We use the same shunt fixture used above, perform OSL calibration, attach the LC circuit to the fixture, and do a scan. We then open Functions→RLC Analysis, which opens the dialog shown in Figure 4.



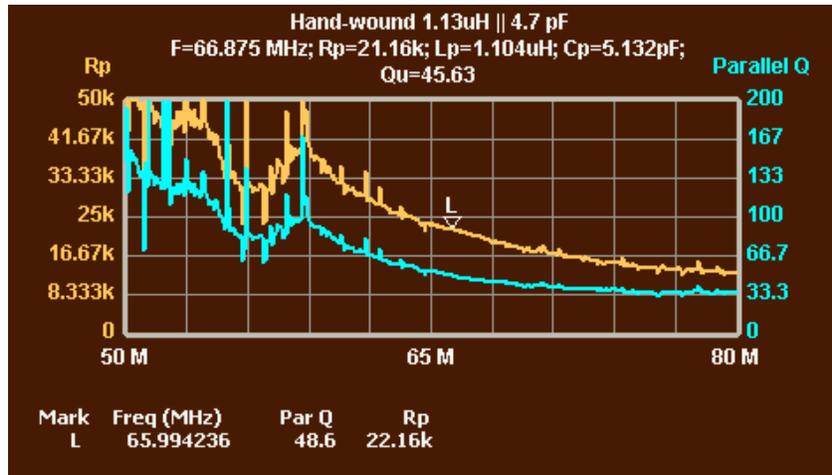
**Figure 4—RLC dialog in Reflection Mode  
(Current dialog is somewhat different.)**

The dialog shown in Figure 4 is an old version. The current version has two changes:

1. The lossy inductor option has been deleted. It was complicated.
2. The option is provided to analyze the circuit at two frequencies pre-marked by the user, as opposed to the method shown in the above dialog, which is referred to as the Slope method.

Here we will look at the results produced from use of the Slope method, which is still available in the current version. The results at the resonant frequency are shown in the text box of Figure 4. They differ somewhat from those shown in Figure 3, and may be more accurate, due to the use of OSL calibration.

The major difference between this analysis and that of Transmission mode is that values are produced for all frequencies in the scan, not just for the resonant frequency. Unfortunately, the graph of Q corresponding to Figure 4 no longer exists. However, a similar analysis was done with a different inductor/capacitor combination, this time in parallel (using a series fixture). Figure 5 shows the results.



**Figure 5—Graph of Resistance and Q over Frequency**  
**Different inductor and capacitor than used above.**  
**Title lines 2 and 3 show results from RLC Analysis text box.**

The blue graph in Figure 5 shows the graph of Q that is produced by Reflection mode RLC Analysis. Marker L was near the resonant frequency. The Q graph is labeled “Parallel Q” simply to distinguish it from the “Component Q” graph produced by the simple analysis method first described above. High Q values not close to resonance are somewhat erratic, but the point is still made that the inductor in question in Figure 5 would produce higher resonance  $Q_U$  at lower frequencies. (This is due to the inductor being wound on T-68-6 toroid whose losses start to become large above 50 MHz.) The inductor described in most of this document would have the opposite behavior, with higher Q at higher frequencies, as was shown in Figure 1.

### Series vs. Parallel RLC

We initially added a series capacitor to our inductor to create resonance, and analyzed it using a shunt fixture, which is optimal for measuring the small impedances that occur near series resonance. The example in Figure 5, however, used a capacitor in parallel with the inductor, and used a series fixture, which is optimal for the large impedances that occur near parallel resonance. The shunt fixture is often more convenient, because it can be built with an SMA connector and the inductor/capacitor combination can be mounted on the back of a mating connector; the components used for calibration can be similarly attached and easily removed. The series fixture requires the components being tested to be directly soldered to the fixture, and the calibration Short and Load must be similarly soldered and unsoldered.

The convenience of the shunt fixture suggests that a series RLC arrangement is preferable. However, that arrangement has a significant disadvantage if we are testing the inductor at frequencies higher than 10% of its SRF. The parallel parasitic capacitance of the inductor causes the apparent value of inductance to rise (because it causes the reactance to rise) as we get anywhere close to the SRF. This distorts the measurement of both L and Q. RLC Analysis of the series combination will separately identify the series L and C components, but the identified L is not the true value of the inductor; it is the value of the inductor distorted by the parasitic capacitance. Only the series capacitor is separated out.

If we use a parallel arrangement of the inductor and capacitor, the parasitic capacitance of the inductor is effectively merged with the parallel capacitor. The L and C values produced by RLC Analysis will then accurately reflect the true underlying inductance. The parasitic capacitance will now be distorting the C value rather than the L value, in that the measured C will not be the true value of the attached capacitor, but will be the sum of the capacitor's value and the parasitic capacitance of the inductor. For present purposes, we don't care about determining the actual value of the capacitor.

Therefore, we obtain the best accuracy in the measurement of L, and therefore Q, by placing the capacitor in parallel with the inductor and using a series fixture. This increased accuracy is significant only if we are testing at frequencies above 10% of the inductor's self-resonant frequency. We can get an approximate idea of the SRF by doing an uncalibrated Transmission mode scan of the inductor with either a series fixture (resonance will produce a notch) or a shunt fixture (resonance will produce a peak). Alternatively, we can measure the inductance by the simple method used to create Figure 1 and see whether the inductance is rising significantly at the frequency of interest. If so, then parallel RLC Analysis is the way to go.