

Determination of Dielectric Constant Of Printed Circuit Boards

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Introduction

The dielectric constant of PCB material is important in determining the trace width required to produce a characteristic impedance of 50 ohms, or any other desired impedance. The most commonly used material, FR-4, has a dielectric constant that may vary widely between manufacturers or batches, and also varies over frequency. It is useful to have a method to determine the dielectric constant for a particular board in order to properly use that board.

Technically, we are seeking the *relative* dielectric constant (ϵ_r); i.e. the value relative to the dielectric constant of a vacuum. But we here just refer to it as the dielectric constant (ϵ). The dielectric constant is sometimes referred to in the literature as E_r , K or D_k .

A related quantity, relative permittivity, contains a real and imaginary part, the real part of which is the dielectric constant and the imaginary part of which represents a loss factor. The ratio of that loss factor to the dielectric constant is the dissipation factor, also known as $\tan(\delta)$. Here we are concerned primarily with the dielectric constant, though the dissipation factor, which has an effect on the Q of resonance, also has some relevance.

We here present a largely non-destructive way to measure the dielectric constant for a particular board, using measurements of resonant frequencies. But first we present a simple method using capacitance, which is useful in some circumstances.

All measurements were made with a build of Scotty's Modular Spectrum Analyzer, also known as the MSA, which combines a spectrum analyzer and Vector Network Analyzer with a frequency range of approximately 100 KHz-3 GHz. The capacitance method can also be used with a capacitance meter (such as the AADE meter). The resonance method using transmission measurements does not measure phase and therefore can be performed with a spectrum analyzer with tracking generator, or even with an adjustable signal level and a power meter.

Capacitance Method

The capacitance of two parallel plates (the copper planes of a PCB) with an area of A square inches and D inches apart, is determined by the dielectric constant of the material between them. If we measure the capacitance C (pF), the dielectric constant is:

$$\epsilon = \frac{C \cdot D}{0.225 \cdot A} \quad (\text{Eq. 1})$$

There is some error in this formula due to fringe capacitance. However, it was found that cutting a 0.062" FR-4 PCB into pieces as small as 1" x 2" (using a shear, so there was no loss of material) resulted in pieces whose total capacitance equaled that of the original, so the increase in total edge length had minimal effect.

The major shortcoming of this formula is that a PCB several inches long/wide does not make a very good “lumped” component except at fairly low frequencies, generally well below the frequencies at which we would be using microstrip whose exact impedance is critical. However, it turns out that the PCB can be well modeled as a lumped capacitor in series with a small inductance. Figure 1 shows the capacitance measurement of a 1.1” square piece of 0.020” FR-4, made by doing a reflection scan using an SMA connector that slid onto the side of the board.

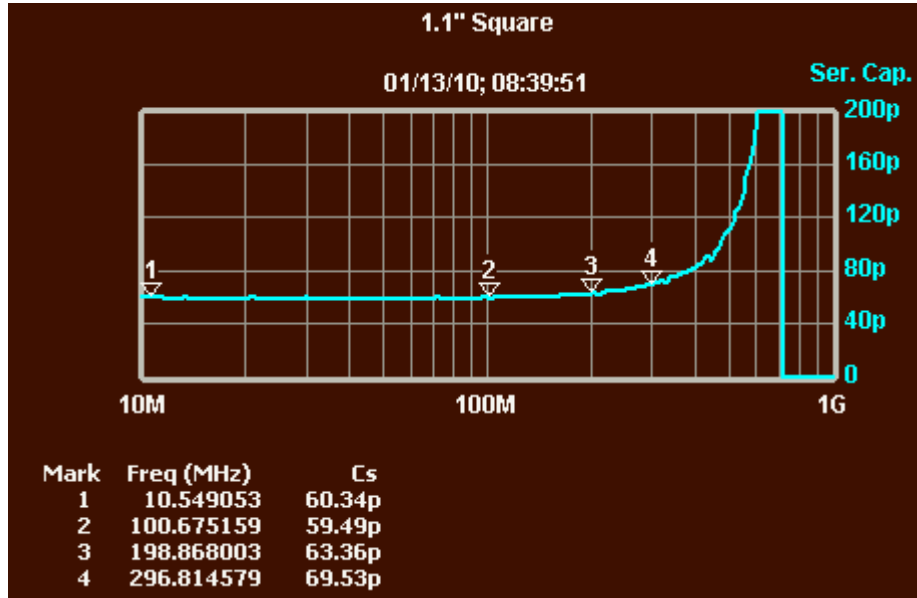


Figure 1—Capacitance of PCB

The capacitance is fairly stable to 100 MHz, but then starts to increase as the resonance at 700 MHz is approached; beyond resonance the capacitor becomes an inductor. In reality, the underlying capacitance does not undergo these transformations; the effective series inductance makes the capacitance appear to change. The problem is, we are looking at a series LC combination as though it were a capacitor, which it is not.

One way to deal with the effect of the inductance is to reduce it. Placing the test connector in the center of the board (with the connector body well soldered to one side and the center pin extending through a small hole and soldered to the other side) will reduce the amount of the inductance, and will be even more effective if the board is cut to a circular shape with the connector at the center.

An alternative is to mathematically remove the effect of the inductance. The MSA has a function called RLC Analysis, which models the board as an RLC combination (here, a series combination), and separately identifies the underlying capacitance and inductance. Figure 2 shows the result of this analysis.

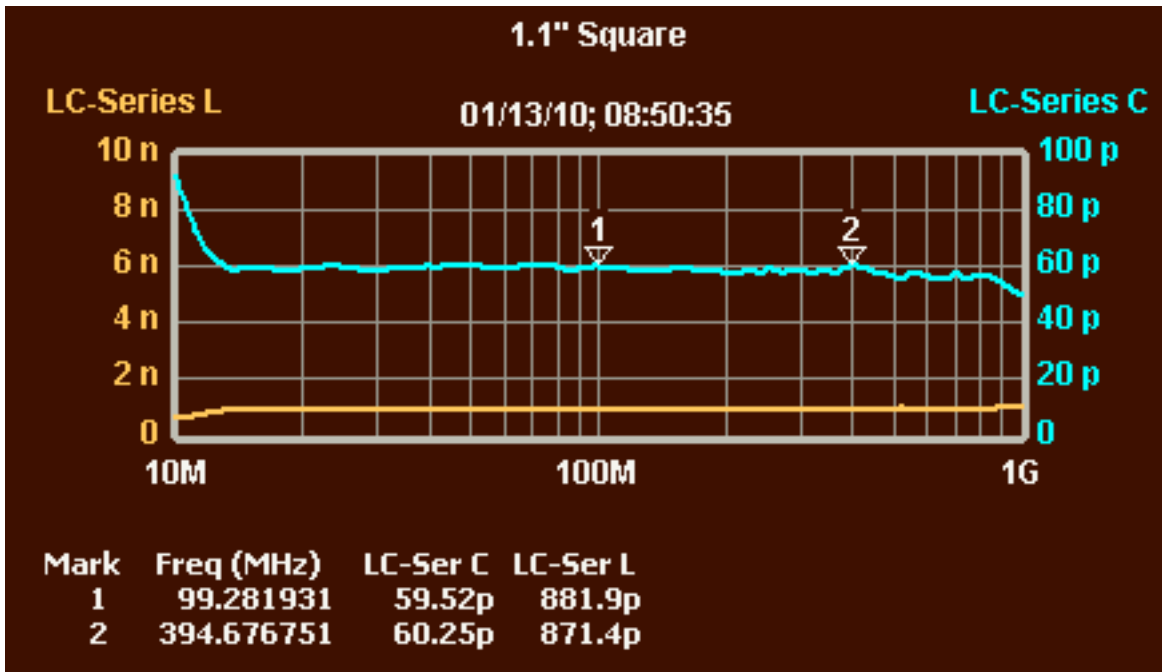


Figure 2—L and C values separately broken out

RLC analysis uses several points to compute some slopes, so it gets distorted near the low and high end where it runs out of points. At 100 MHz, the C values are essentially the same in Figures 1 and 2, but beyond that Figure 2 is able to identify relatively stable values for the underlying L and C values. Note that there is gradual decline in the capacitance (though marker 2 happens to have landed on a small bump), which will translate into a decline in the calculated dielectric constant. For a capacitance of 60 pF, the calculated ϵ is 4.4, though this board was roughly cut, so the stated dimensions are not precise.

The thickness of PCB is not always well specified. If we want to construct a microstrip of a specific impedance, we need to know the thickness as well as the dielectric constant. The capacitance method can be used to determine the thickness of a PCB if we already know the dielectric constant (which perhaps we determined by the resonance method):

$$D = \frac{0.225 \cdot A \cdot \epsilon}{C} \quad (\text{Eq. 2})$$

Resonance References

The resonance method utilized here was suggested by three sources:

1. L. S. Napoli and J. J. Hughes, "A Simple Technique for the Accurate Determination of the Microwave Dielectric Constant for Microwave Integrated-Circuit Substrates", *IEEE Transactions on Microwave Theory and Techniques*, July, 1971.
2. Howell, "A Quick Accurate Method to Measure the Dielectric Constant of Microwave Integrated-Circuit Substrates", *IEEE Transactions on Microwave Theory and Techniques*, March, 1973.
3. Wang, "Determining Dielectric Constant and Loss-Tangent in FR-4", *UMR EMC Laboratory Technical Report TR-00-1-041*, March, 2000.

These articles each discuss methods of treating the substrate as a resonant cavity, and determining dielectric constant from the resonant frequencies of such cavities. They are referred to below as “Napoli/Hughes”, “Howell” and “Wang”.

In addition, several test methods are set forth at the IPC web site:

<http://www.ipc.org/ContentPage.aspx?pageid=ELECTRICAL-TEST-METHODS>

Section 2.5.5.6 of the above web page describes the resonance method; section 2.5.5 describes the capacitance method.

Cavity Math

In an essentially two-dimensional cavity with length and width but very small height, many resonances are possible. The simplest resonances are at frequencies where the half-wavelength equals the length or the width, but there are many other possibilities. Each resonance corresponds to an integral “wave number” or “mode number” in each direction (length and width). If p is the mode number in the length (L) direction and q is the mode number in the width (W) direction, then the resonant frequency for a given p and q is:

$$f = \frac{c}{2\pi \sqrt{\mu\epsilon}} \sqrt{\left(\frac{\pi \cdot p}{L}\right)^2 + \left(\frac{\pi \cdot q}{W}\right)^2} \quad (\text{Eq. 3})$$

Where c is the speed of light, μ is permeability (for PCB dielectrics, this is 1), and ϵ (epsilon) is the dielectric constant. Each of p and q can have any non-negative integral value, except both cannot simultaneously be zero.

To measure L and W in inches, and f in MHz, we can convert the above formula into this form:

$$f = \frac{983.6}{2\sqrt{\epsilon}} \sqrt{\left(\frac{12p}{L}\right)^2 + \left(\frac{12q}{W}\right)^2} = \frac{5902}{\sqrt{\epsilon}} \sqrt{\left(\frac{p}{L}\right)^2 + \left(\frac{q}{W}\right)^2}$$

(Eq. 4) (f in MHz, L and W in inches)

(Note: the speed of light is 983.6 million feet per second.)

These equations look intimidating, but there are two situations where their meaning is very straightforward. If q is 0, then there is no resonance in the width direction, and the resonance is entirely in the length direction. The equation then looks like this:

$$f = p \cdot \frac{983.6 \times 12}{2L\sqrt{\epsilon}} \quad (\text{Eq. 4A—when } q=0)$$

Everything on the right side of the equation except p represents the frequency whose half-wavelength is 12/L feet. Therefore, the various resonant frequencies are integral multiples (that integer is p) of that half-wavelength frequency.

Similarly, if p is zero, the resonance is entirely in the width direction, at multiples (q) of the fundamental frequency for which half-wavelength equals the width.

If we treat length as the longest dimension, we know that the lowest frequency resonance will be in the length direction with the length equal to a half-wavelength, so p=1 and q=0. If ε is fairly constant, there will be resonances near multiples of that frequency, with p=N and q=0. There will be a similar set of resonances purely in the width direction, with p=0 and q=N. Mixed in with these resonances along the length or width, are resonances that are operating in both directions at once. For those, neither p nor q is zero.

If the length and width are equal, the first two resonances will fall at the same frequency. If the width is smaller than the length but is at least 58% (one over the square-root of 3) of the length, the second resonance will be the fundamental resonance in the width direction (p=0, q=1) and the third will be a combined resonance with p=q=1. For narrower widths, there are all sorts of possibilities for the order of resonances. For very narrow widths, the first several resonances will be entirely in the length direction with p=1,2,3... and q=0.

If we measure the resonant frequency f for a given p and q, we can calculate the dielectric constant as:

$$\epsilon = \left(\frac{5902}{f} \right)^2 \left[\left(\frac{p}{L} \right)^2 + \left(\frac{q}{W} \right)^2 \right] \quad (\text{Eq. 5}) \text{ (f in MHz, L and W in inches)}$$

(If L and W are in cm, the factor 5902 must be increased to 14990.)

The fact that there are so many possible resonances lets us measure ε over a broad range of frequencies with a single PCB.

For the utmost in precision, Howell and Wang set forth a correction factor to account for losses, based on the Q of the resonance. The measured frequency is adjusted per the following formula, and the adjusted frequency is then used in Eq. 5.

$$f_{ADJ} = \frac{f_{MEAS}}{\left(1 - \frac{1}{2Q} \right)} \quad (\text{Eq. 6})$$

Note that for Q=50 (the approximate value for FR-4), the denominator is 0.99, which increases f by 1%, and therefore reduces the calculated ε by 2% (because frequency is squared in Eq. 5). For Q over 100, there probably is no point bothering with the adjustment.

Application to Printed Circuit Boards

A double-sided PCB might be treated as an open-sided cavity, so for the moment assume that it is proper to do so. If one side is treated as the ground plane, the signal can be loosely coupled to the other side, creating a signal that travels primarily between the two planes, inside the PCB dielectric. If we measure certain resonant frequencies of this cavity, we can calculate the dielectric constant from Eq 5. We first do this in transmission mode. A piece of double-sided

FR-4 is cut to 4" x 5". SMA connectors are soldered in two opposite corners. They can be soldered to one side with their legs broken off and the center pin extending through a hole to the other side, but not actually soldered to the other side. Or, for thin material, they can be mounted edge-wise with the center pin projecting over but not touching the top copper plane. The stimulus signal is attached to one connector and the response is taken from the other. Figure 3 shows the test setup (attenuators are optional).



Figure 3—PCB Corner-to-Corner Transmission Test Setup

For FR-4, the connectors were vertically mounted without legs, with the center pin projecting through the board but not attached to the other side.

Schematics for this test method and others described below are set forth in Appendix B. Appendix C contains photos of some methods of attaching the connectors. Figure 4 shows the results of the corner-to-corner transmission test.



Figure 4—Transmission of FR-4 from corner to corner

Data below -80 dB is primarily noise.

There are three resonances shown in Figure 4. Marker 1 shows the resonant frequency for which the half-wavelength is the longer dimension. Marker 2 shows the resonant frequency for which the half-wavelength is the shorter dimension. If we treat length as the longer dimension, and label the modes as $M(p,q)$, then marker 1 is $M(1,0)$ and marker 2 is $M(0,1)$. Marker 3 is the first mode that involves resonance occurring in both directions and is $M(1,1)$.

This notation for modes differs from the typical notation TE_{101} , etc. For our purposes, all we care is that each mode is identified by two integers, p and q , which integers we can plug into Eq. 5. We are not concerned with the details of the signal propagation.

If we apply Eq. 5 to each of these resonances, using the f , p and q values for each, we get a dielectric constant of 4.4 for markers 1 and 2 and 4.3 for marker 3. The adjustment for Eq 6,

assuming a Q value of 50, would reduce each of these values by 0.1, to 4.2 and 4.3 respectively. (Q was measured separately, by finding -3 dB points, and was near 50.) Those are very plausible values for FR-4 at these frequencies, though its dielectric constant can vary so much between boards that we can't make any judgment about the accuracy of our method from this data.

We can also measure transmission in another way: run the signal through the board from one plane to the other with neither side being grounded. This works best if done in a corner. Exhibit C shows that this can be done with a single probe consisting of two parallel semi-rigid coax cables with their shields soldered together at the end, and their center conductors protruding slightly. Place the PCB between the center conductors, and twist as necessary to get contact on each side. Figure 4A shows the results.

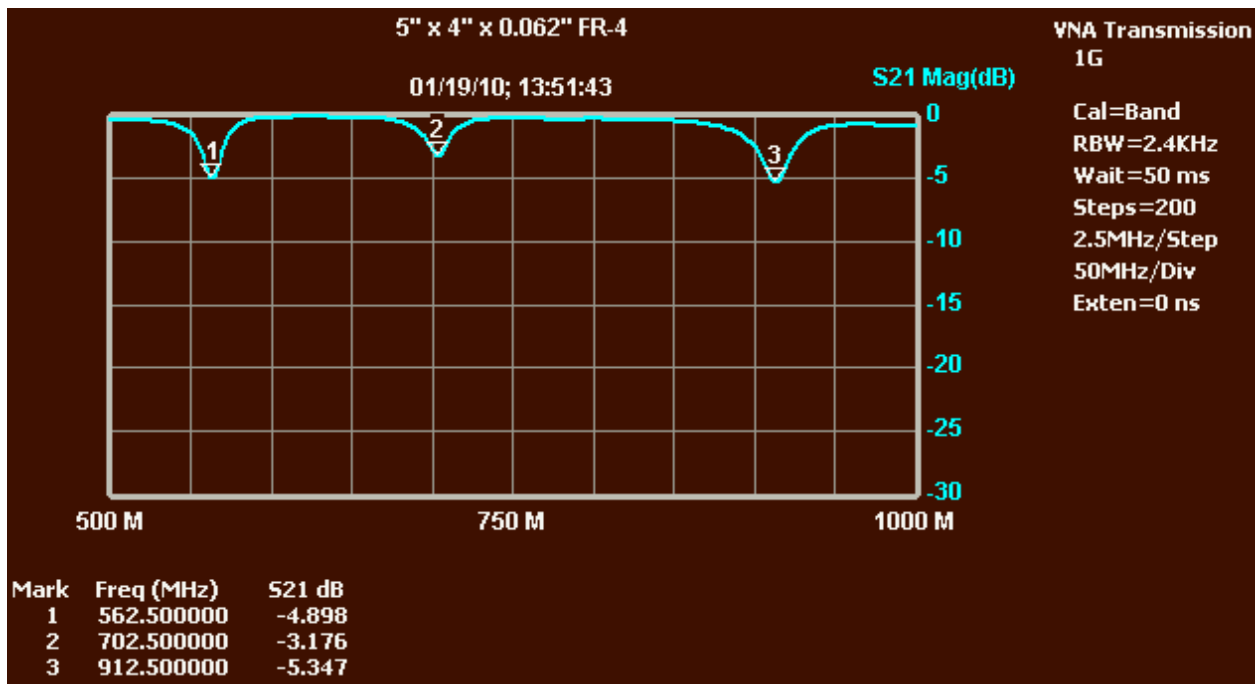


Figure 4A—Plane-to-Plane Transmission
Input and output occur on opposite sides of the same corner

Figure 4A shows essentially the same resonant frequencies as Figure 4, but we now have transmission dips instead of transmission peaks.

Instead of measuring transmission, we can place a single PCB-mount SMA connector sideways on the corner of the board, with the center pin touching the top and the connector legs touching the bottom (ground plane), and then measure reflection. Figure 5 shows the results.

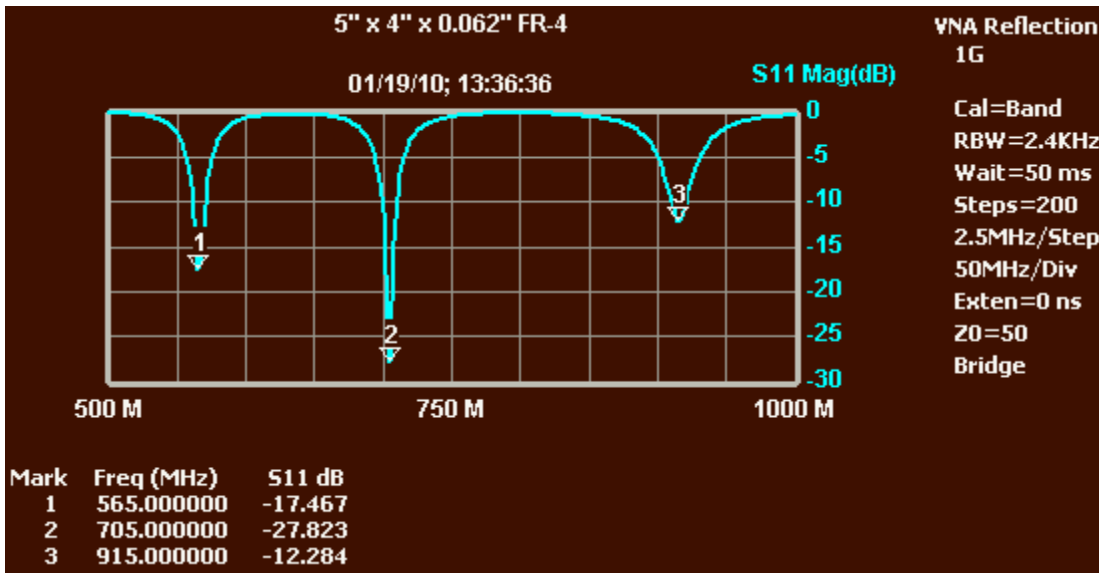


Figure 5—Reflection of PCB

The resonances shown in Figure 5 are essentially the same as those in Figures 4 and 4A. Any of these methods is suitable to locate the resonances. Corner-to-corner transmission measurements allow measurement of Q if desired but require that two connectors be coupled to the PCB. Either transmission method utilizes fairly simple calibration. Transmission from plane to plane requires only a single probe, and simple calibration. Reflection measurements require only one connector, but require calibration scans for the Open, Short and Load.

Reflection and Plane-To-Plane transmission are really measuring the same thing. The MSA can convert S11 readings into S21 (menu Functions->Generate S21), by simulating the impedances indicated by the S11 readings, placed in a transmission fixture. Figure 5A shows the result of such a conversion, in which the impedance is hypothetically placed in a “series” fixture, exactly as the PCB is placed in Plane-to-Plane transmission.

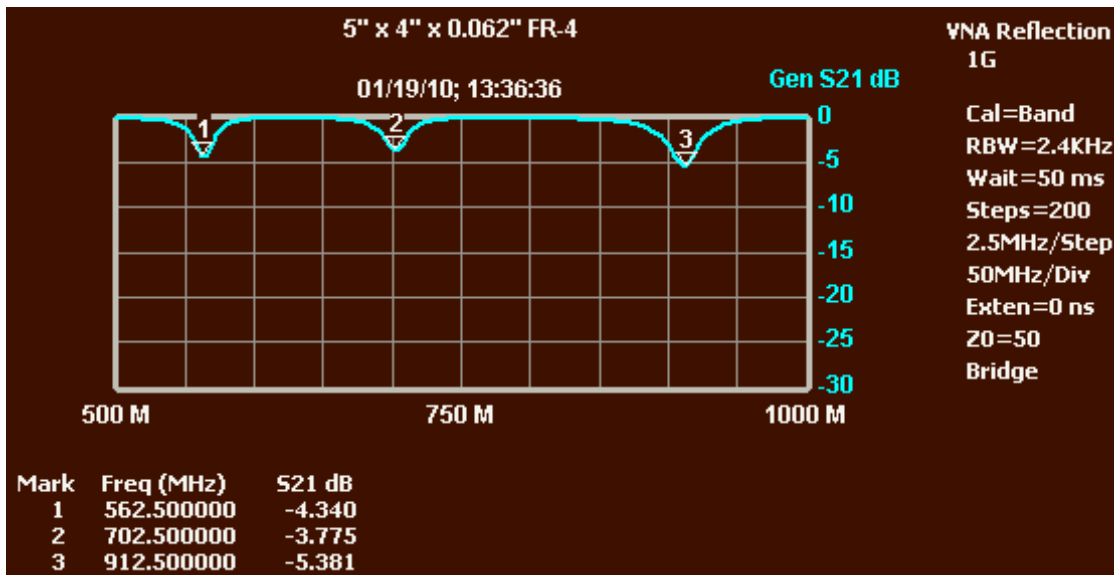


Figure 5A—Reflection converted to Plane-To-Plane Transmission
Should in theory match Figure 4A

Theoretically, Figure 5A should match Figure 4A. The resonances match, but there are small differences in transmission levels, which depend somewhat on placement of connectors.

We have so far treated the PCB as a cavity. The side walls have no conductors, so it is an open cavity. If we seal the sides of the board with copper tape, we will have a closed cavity. While Howell indicates that we should be able to measure transmission of this closed cavity, we got no meaningful response when measuring the corner-to-corner transmission of such a closed cavity. We simply couldn't come up with a good method to get the signal inside the cavity. With a connector in the center of the board used to measure reflection, we can see a couple of resonances up to 3 GHz, but none that seem consistent with the above equations.

So we will stick with the open-sided PCB. It might be questioned whether this is truly acting as a cavity, or simply as a sheet of copper over a ground plane, but in fact we will see from measurements of GML 1000 that the two seem to be pretty much the same thing. Indeed, in several of the later measurements, grasping the PCB on both sides with fingers had minimal impact on the measurement, suggesting that all the "action" is occurring in the interior of the PCB.

Tests with GML 1000

GML 1000 was a precise PCB material that is no longer manufactured. Its dielectric constant was specified as 3.05 ± 0.05 , at both 2.5 GHz and 10 GHz. Its dissipation factor is about 0.005. We will assume the same spec applies below 2.5 GHz. This precise specification gives us a good reference for comparison of our own measurements.

A 5" x 4" piece of double-sided GML 1000 was fit with SMA connectors on two opposing corners. Each connector was mounted edge-wise, with two legs soldered to the bottom plane and the center pin extending over, but not attached to, the top copper plane. This provided loose coupling of the signal to the top plane. Transmission was then measured. This method has the advantage that no holes are drilled; other than a bit of solder in the two corners, the PCB is not harmed by the test. Three separate scans were conducted in order to cover the range up to 3 GHz, using the 1G, 2G and 3G frequency bands of the MSA.

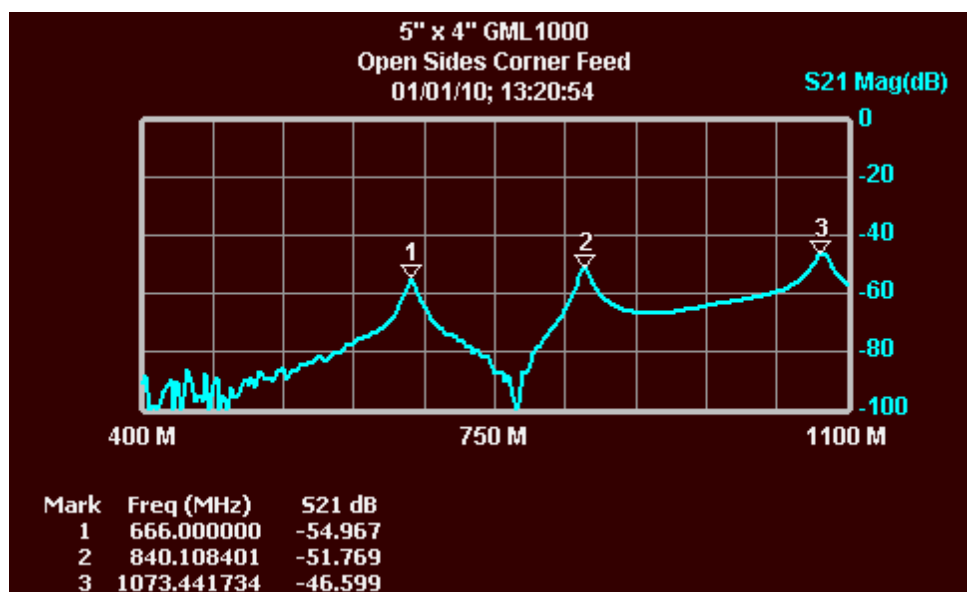


Figure 6—Low Frequency GML

Figure 6 shows the first 3 modes: M(1,0), M(0,1) and M(1,1), where the first mode number (p) refers to the longer dimension. Thus, M(1,0) represents the frequency with a half-wavelength equal to 5". These three frequencies are always easy to identify, except that if the PCB is nearly square, resonances 1 and 2 will merge to a single peak. Q was measured at markers 1 and 2, by locating -3 dB points, and was approximately 90. This is lower than one would expect purely from the dissipation factor of 0.005, suggesting there are some additional losses that occur at the board edges due to radiation or other factors. However, the loss is still much less than for FR-4, so the peaks in Figure 6 are sharper than those in Figure 4.

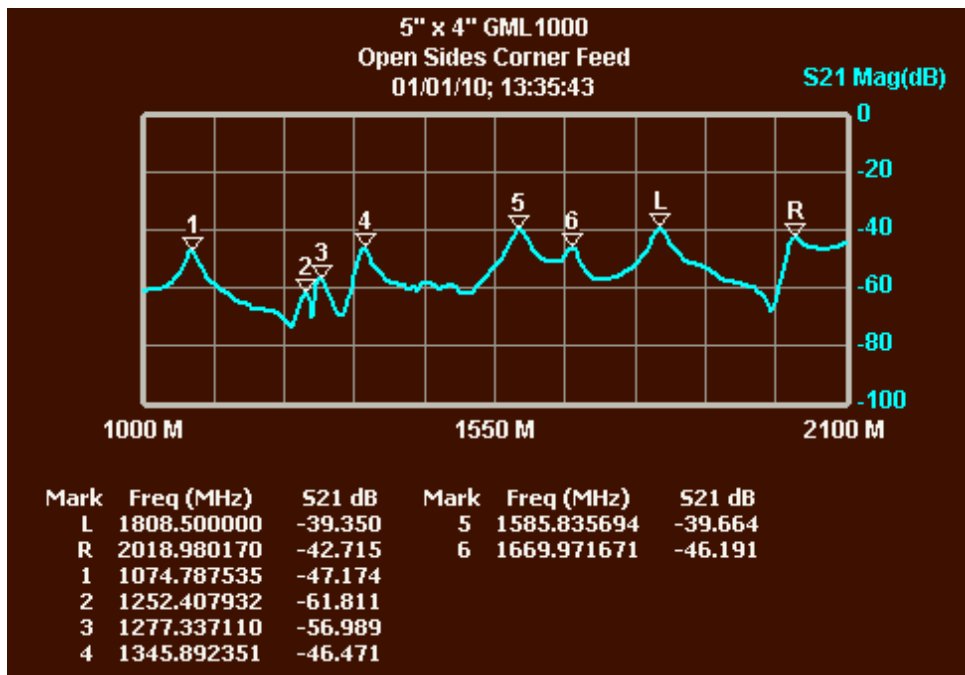


Figure 7—Mid Frequency GML

Figure 7 is taken with the MSA set to the 2G band, and shows many resonances.

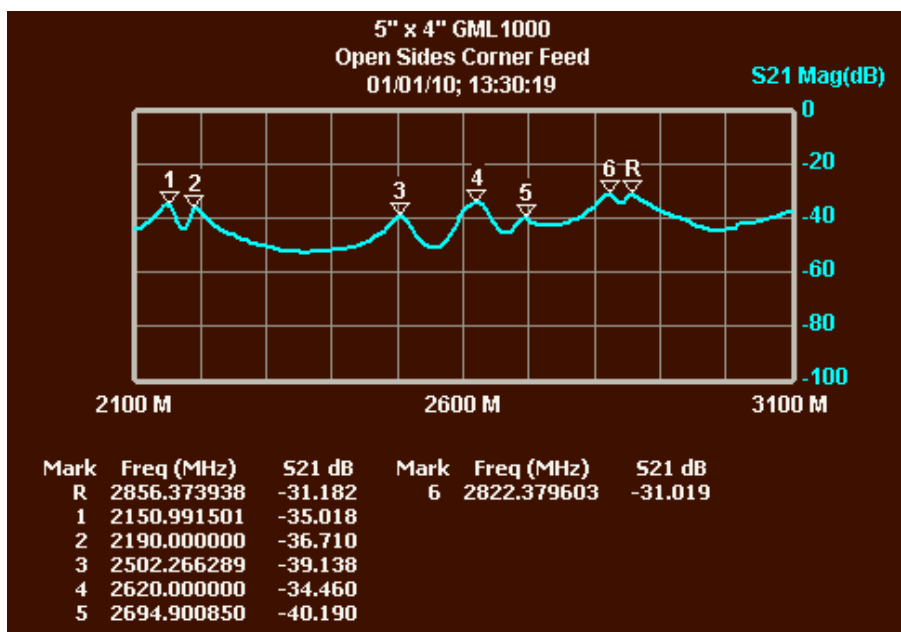


Figure 8—High Frequency GML

Figure 8 is taken with the MSA set to 3G band, and also contains many resonances. They don't show up as prominently as those at lower frequencies (largely because the connector capacitive coupling becomes stronger at higher frequencies), but it will be seen that they are all very close to where they should theoretically be.

To match the resonances to specific modes, we first calculate the dielectric constant from the first three resonances, which are easy to identify. They provide values of approximately 3.1, so we assume that as the actual value and calculate where all the resonance peaks should occur. We then match the actual peaks to the calculated peaks as best we can. The matches are shown in Table 1.

Table 1—Calculated and Actual Resonant Modes of 5" x 4" GML 1000

Assumed ϵ = 3.1
 Length= 5 in
 Width= 4 in

Sorted by Mode					Sorted by Freq				
p	q	Calc Freq	Meas Freq	Calc ϵ	p	q	Calc Freq	Meas Freq	Calc ϵ
0	1	838	840	3.09	1	0	671	666	3.14
0	2	1677	1669	3.13	0	1	838	840	3.09
0	3	2515	2502	3.13	1	1	1074	1073	3.10
1	0	671	666	3.14	2	0	1341	1345	3.08
1	1	1074	1073	3.10	2	1	1582	1585	3.08
1	2	1806	1808	3.09	0	2	1677	1669	3.13
1	3	2603	2620	3.06	1	2	1806	1808	3.09
2	0	1341	1345	3.08	3	0	2012	2019	3.08
2	1	1582	1585	3.08	2	2	2147	2151	3.09
2	2	2147	2151	3.09	3	1	2180	2190	3.07
2	3	2851	2856	3.09	0	3	2515	2502	3.13
3	0	2012	2019	3.08	1	3	2603	2620	3.06
3	1	2180	2190	3.07	3	2	2619	2620	3.10
3	2	2619	2620	3.10	2	3	2851	2856	3.09

The left part of the table lists the resonances by mode; the right side lists them by frequency. The left two columns in each part are the p and q values, p being the mode number for the Length dimension. The third column is the frequency at which the mode should appear if $\epsilon=3.1$. This column is used to match the mode to a specific peak; the actual frequency of the matched peak is in the fourth column. Finally, from the measured frequency we calculate ϵ , which is listed in the final column of each table. The measured frequency 2620 MHz appears twice in the table, because it is ambiguous as to whether it matches M(1,3) or M(3,2); but either way the ϵ value calculated for that frequency is very good.

Table 1 shows that we can match most of the resonant peaks to specific resonant modes, and the calculated dielectric constants are very consistent, varying from 3.06 to 3.14. Recall that the actual value for GML 1000 is anywhere from 3.0 to 3.1. Our measurements are therefore probably about 1.5% high. If we make the Eq. 6 adjustment for Q, based on the Q measurements for the lowest two resonances, we would adjust the frequencies upward by dividing by $1-1/180$, or 0.994, a 0.6% increase, which would reduce the calculated ϵ by twice that (because frequency is squared in Eq. 5), or 1.2%. This would nearly eliminate the error in our results, though the adjustment is so small that it may not be worth the effort.

Howell suggests that measuring a substrate with open sides leads to error and erratic results due to radiation losses. However, our results, and those of Napoli/Hughes, are much more self-consistent than the ones he obtained for open-sided measurements. Therefore, the small adjustment for Q would seem to deal effectively with the radiation losses.

Efforts to measure transmission with closed-sided boards (sealed with tape) were not successful. Wang describes a method for doing so with FR-4 using reflection measurements, but his method of coupling the signal to the cavity created by sealing the board is not clear, and efforts to duplicate his work produced very few resonances and ϵ values that seemed 20-30% in error.

FR-4's Varying Dielectric Constant

Applying the technique used in Table 1 to match resonances to specific modes is a little trickier for FR-4. Its dielectric constant varies with frequency, so calculating expected resonances from the ϵ value determined for the first several resonances may not provide sufficiently precise data to determine which resonance matches which mode. The assumed value may have to be tweaked to get good matches in the 2G and 3G ranges.

The task of matching is much simpler for a long, narrow board, because the initial resonances will all relate to the longer dimension and are very easy to identify. Table 2 shows several resonances of a 12" x 1.5" piece of FR-4 (0.020" thick).

p	q	MHz	ϵ
1	0	232	4.49
2	0	468	4.42
3	0	708	4.34
4	0	945	4.33
5	0	1190	4.27
6	0	1426	4.28
7	0	1666	4.27
8	0	1902	4.28

Table 2—Resonances of 12" x 1.5" FR-4

At the high frequency end, there were other resonances mixed in, but the above modes were easy to pick out. Table 2 also illustrates the declining value of ϵ for FR-4 as frequency increases. Some studies have also shown that FR-4 exposed to high humidity will have a greater decline at high frequency. Note that with a wider board, if you tried to locate the final resonance in Table 2 by looking for the 8th multiple of 232 MHz (1856 MHz), the actual value of 1902 MHz (due to the declining ϵ) is far enough away that there might be other nearby resonances that would confuse the matching process.

It is possible that long, narrow boards will show some edge effects that distort the measurements, so the above test was partially repeated with boards that were 1 inch, 2 inches and 7.8 inches wide. Table 3 shows the results:

1 Inch				2 Inch				7.8 Inch			
p	q	MHz	ϵ	p	q	MHz	ϵ	p	q	MHz	ϵ
1	0	233	4.46	1	0	232	4.49	1	0	233	4.46
2	0	470	4.38	2	0	470	4.38	2	0	470	4.38
3	0	703	4.40	3	0	708	4.34	3	0	707	4.35
4	0	940	4.38	4	0	945	4.33	4	0	945	4.33

Table 3—Narrower and Wider Boards (all 12” long, 0.020” thick)

(The 7.8 inch wide board showed many additional resonances matching to modes with $q>0$.)

The 2-inch wide board matched the 1.5-inch board more closely than the 1-inch board did, suggesting there are some small edge effects that kick in at the 1-inch width but are having minimal effect at the wider sizes. The additional width increase from 2 to 7.8 inches had virtually no effect. Conceivably, the width where edge effects show up will increase with thicker boards (whose edge fields perhaps extend further away), so if narrow 0.062” boards are tested, it is a good idea to do a one-time test of several widths to check for consistency.

Tests With Duroid 5880

The test procedure was repeated using Rogers Duroid 5880, which has a very well controlled dielectric constant of 2.2 +/- 0.02. Only the first two resonances were examined, and they were located by using a scan in reflection mode, as shown in Figure 9.

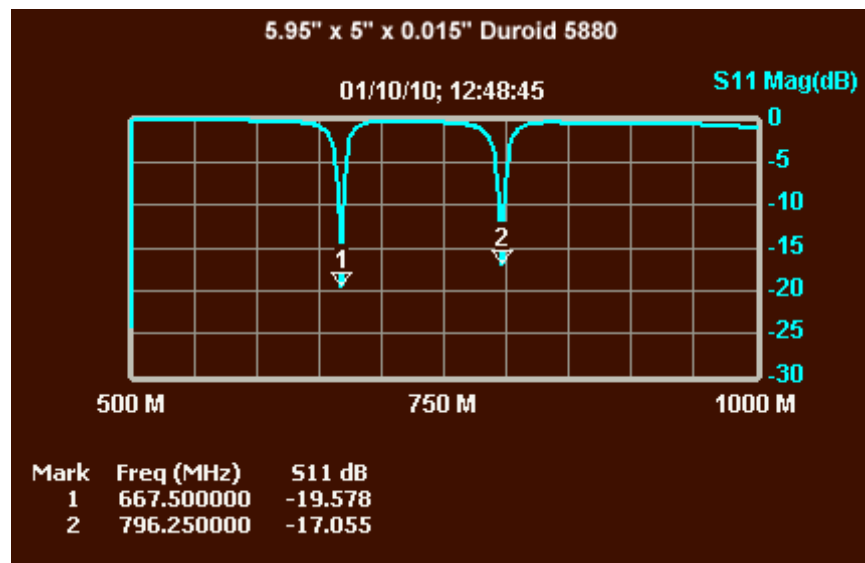


Figure 9—Duroid 5880

Marker 1 is Mode(1,0) and marker 2 is Mode(0,1). Using Eq. 5, the dielectric constant indicated by marker 1 is 2.21 and that indicated by marker 2 is 2.20. Both values are within the specification. Because we used the reflection method, we did not measure Q. But the Duroid dissipation factor is extremely small (0.0003 or so), so PCB losses would be negligible. Radiation losses are still possible, as with the GML 1000, but any adjustment for Q under Eq. 6

would likely affect the calculated dielectric constant by less than 1%, still leaving the results within the specification.

Single-Sided PCB

A single-sided PCB could be measured by the resonance method if it we could temporarily add a ground plane. One way to do this is to clamp the PCB to a flat piece of aluminum. That was tried with a piece of Taconic TLC-32, with a dielectric constant near 3.2. Table 4 shows the results.

Assumed ϵ = 2.8
 Length= 7.65 in
 Width= 6.05 in

p	q	Calc Freq	Meas Freq	Calc ϵ
0	1	583	542.5	3.23
1	0	461	460	2.81
1	1	744	722.5	2.96
2	0	923	943	2.68
2	1	1091	1100	2.75

Table 4—Single-Sided TLC-32 clamped to aluminum plate

The clamping job was not ideal. The results are somewhat erratic and generally about 10-15% low. A better clamping method would be to place the PCB on aluminum, with a sheet of soft rubber or silicone on the top, and a flat board on top of that. Such a method would provide more uniform pressure over the PCB. However, it is still likely that the measured dielectric constant would be somewhat low, because there would still be a small amount of air between the PCB material and the bottom aluminum plate.

A more accurate, but somewhat destructive, method is to coat the copper-free side with conductive silver paint. Still, the above method provided ball-park results that might at least be useful to identify the type of dielectric material for an unknown board.

Taconic RF-43

Appendix A shows the results of additional tests on Taconic RF-43, a PTFE-based substitute for FR-4. The board was provided gratis by Taconic, who certified that the dielectric constant is 4.4. That is exactly the value produced by an average of measurements at 15 resonances.

Another set of tests was performed on Epsilam-10. The detailed measurements are not shown here, but produced results consistently in the area of 9.5. These tests are the only ones that seemed to show significant error, which may have something to do with the very high dielectric constant. Or, it is possible that these particular boards were simply out-of-spec. They were an eBay purchase, and there may be a reason they were available on eBay.

Conclusion

The described method of calculating the dielectric constant from resonant frequencies, using non-destructive transmission measurements, appears to provide good accuracy. When measuring FR-4, accuracy is likely improved by reducing the calculated dielectric constant by 2%, to account for losses, primarily from the material's dissipation factor.

With that adjustment, the error in the calculated dielectric constant is likely to be only 1-2%. For some perspective, the characteristic impedance of a 50-ohm trace calculated for $\epsilon = 3$ will

decrease by only one ohm if ϵ turns out to be 3.15, an error of 5% in ϵ . An error of several percent in the calculated dielectric constant is therefore very acceptable.

APPENDIX A—Tests with Taconic RF-43

Additional tests were done with Taconic RF-43, with a 12" x 18" sheet kindly provided by Taconic. This board was certified to have a dielectric constant of 4.4, and a dielectric thickness of 0.0626 inches. This dielectric constant is close to what would be expected for FR-4, so these tests helped verify that our measurement methods are accurate with dielectric constants in the 4-5 range.

The board was first measured by Plane-to-Plane Transmission. The board was placed in the last probe shown in Appendix C. Transmission through the board is shown below:

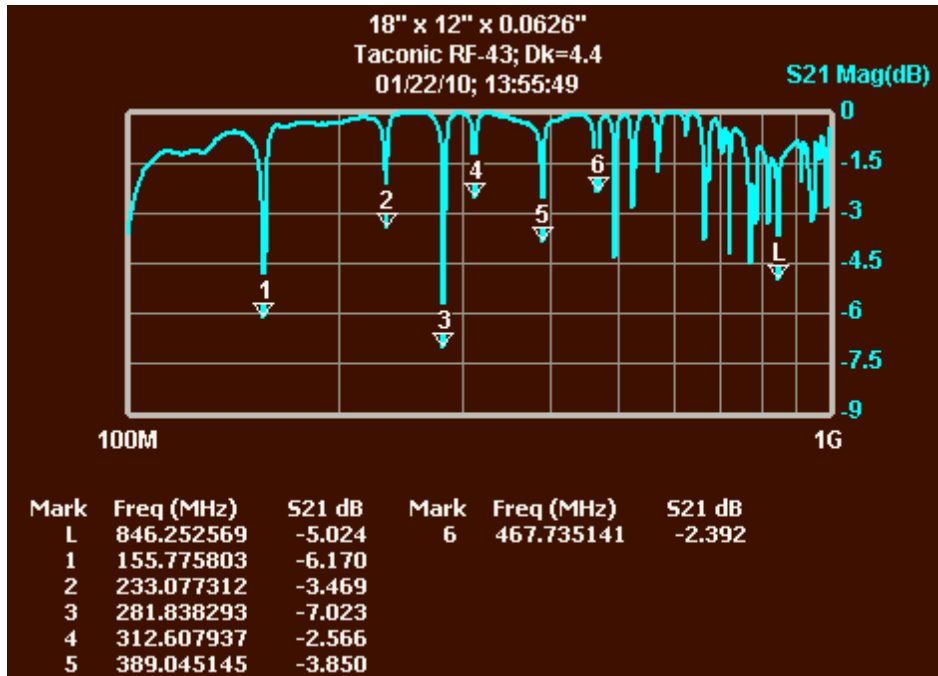


Figure A1--Plane-To-Plane Transmission of RF-43

There are so many resonances that they can't all be marked, but the frequencies of many unmarked ones were also recorded. No measurements were made above 1 GHz with this board, because the large size of the board produces a mass of closely spaced and overlapping resonances.

Transmission was also measured corner-to-corner. SMA connectors were soldered at opposite corners, per the second connector shown in Exhibit C. This provided loose coupling to the signal plane of the PCB. Figure A2 shows the results.

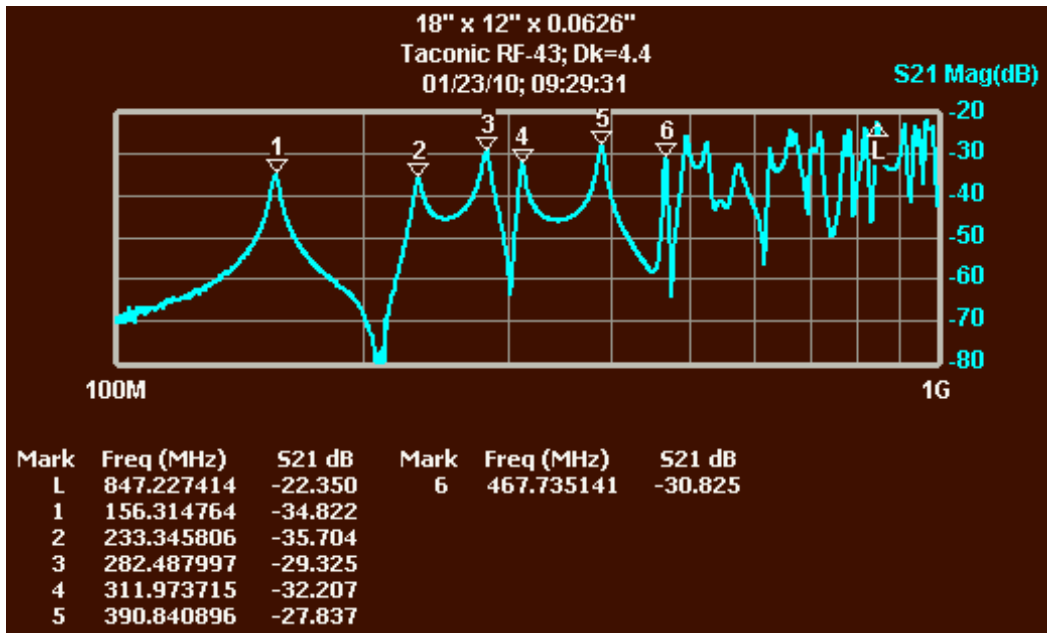


Figure A2—Corner-to-corner transmission fo RF-43

Reflection measurements were also made, as shown in Figure A3.

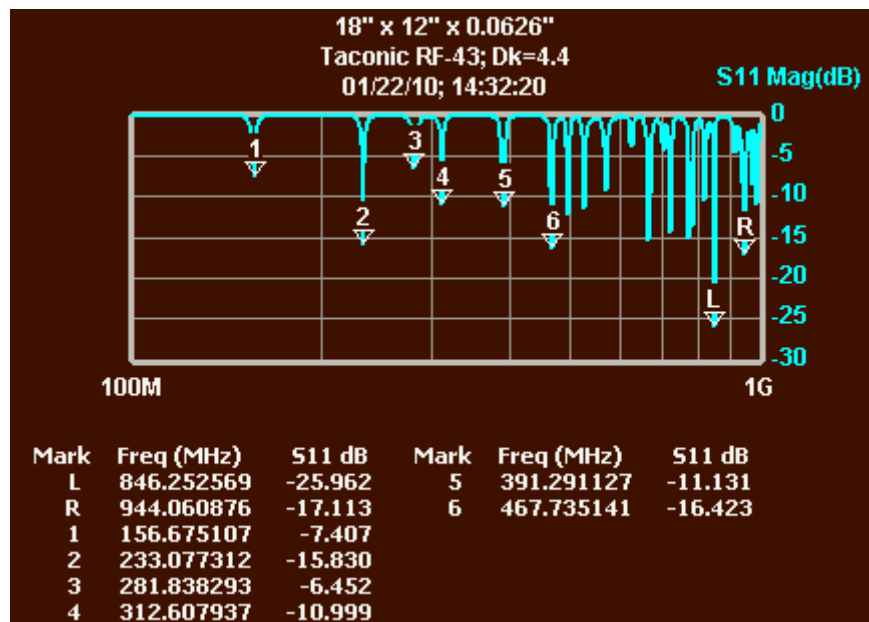


Figure A3--Reflection of RF-43

Both Reflection and Plane-to-Plane Transmission produced very sharp, well-defined dips, at very nearly the same frequencies. Corner-to-corner transmission produced very well-defined peaks at the resonances, closely matching the frequencies identified by the other methods. One advantage of corner-to-corner transmission is that it makes it easy to examine the Q of the resonance, which we will do momentarily.

Putting a number of these frequencies (primarily from Figure A1) into a spreadsheet to calculate the dielectric constant yields the following results:

p	q	Calc Freq	Meas Freq	Calc ϵ
1	0	156	155.8	4.43
0	1	235	233.1	4.45
1	1	282	281.8	4.40
2	0	313	312.6	4.40
2	1	391	389	4.44
0	2	469	468	4.42
3	0	469	468	4.42
1	2	495	495.5	4.38
3	1	525	524.8	4.39
2	2	564	568.8	4.32
3	2	663	664.5	4.38
1	3	721	720.3	4.40
2	3	770	771.8	4.38
3	3	846	846.3	4.39
4	3	942	944	4.37

Table A1--Results for Taconic RF-43

All the results came out within 1% of the certified value of 4.4, and the average comes out exactly to that value.

The capacitance of the Taconic RF-43 board was also measured. The capacitance was 3.48 nF at 1 MHz, indicating a dielectric constant of 4.48. Self-resonance was about 30 MHz, and the capacitance was slowly rising even at 1 MHz, so the “true” capacitance is slightly below the measured value, which would account for the very slight discrepancy between the results by the resonance and capacitance methods.

Effects of Q

Lets’ examine the effects of Q on the resonance frequencies for the RF-43, to determine whether some adjustment should be made under Eq. 6. We adjusted the probes used for corner-to-corner transmission to reduce the coupling, and zoomed in on the resonance at 525 MHz.

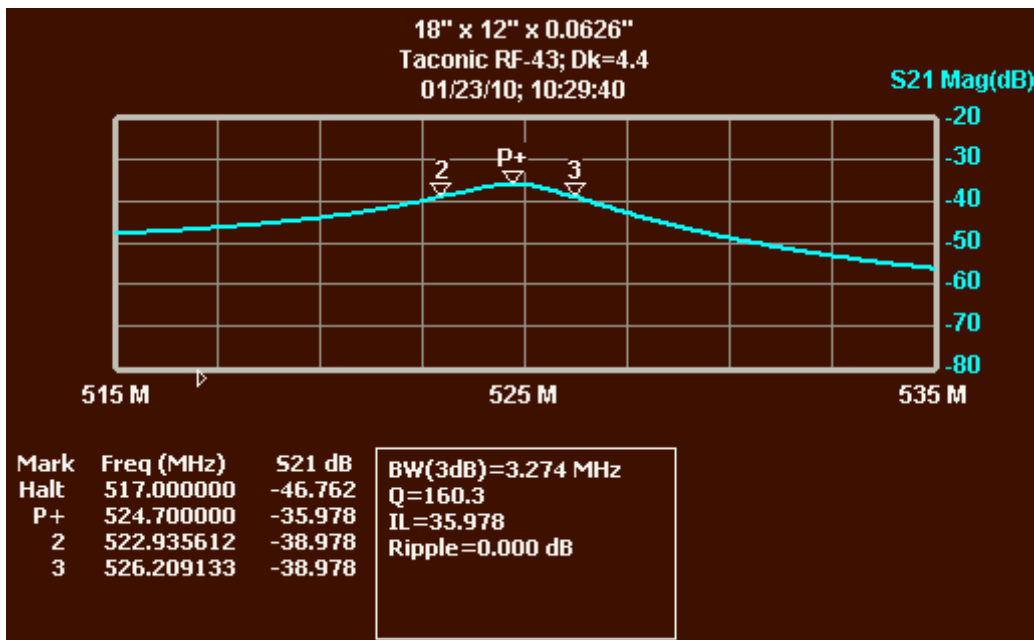


Figure A4—Corner-to-corner transmission with reduced coupling

We used Filter Analysis to automatically place the peak marker and -3 dB points in Figure A4, and to calculate Q, which came out to 160. The specified dissipation factor for RF-43 is 0.0035. If that were the only source of loss, we would expect Q of 286. However, there is some amount of radiation loss, and the probes load the resonance to some extent. Even using this Q value, however, the adjustment of Eq. 6 would be very small—about 0.6%.

Let's look at what happens if we directly couple the probes to the board when performing corner-to-corner transmission. Figure A5 shows the results.

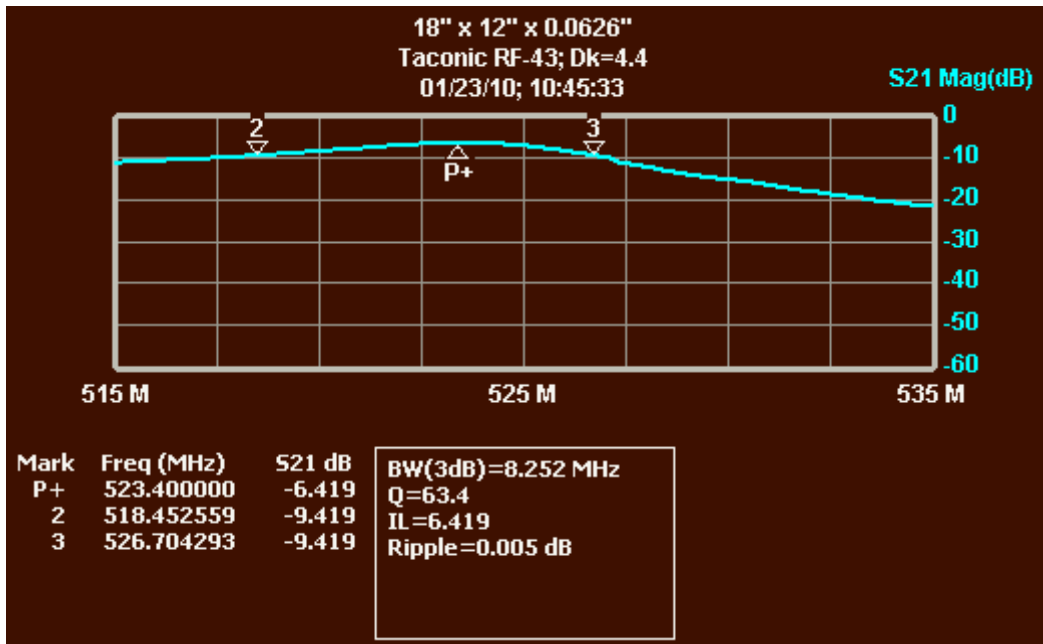
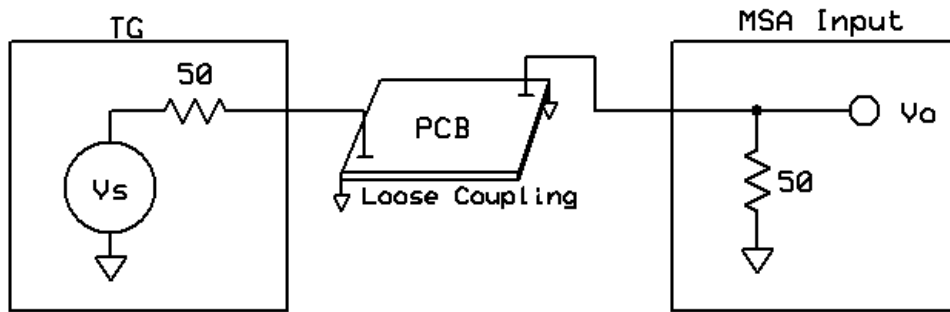


Figure A5—Corner-to-corner transmission with direct coupling

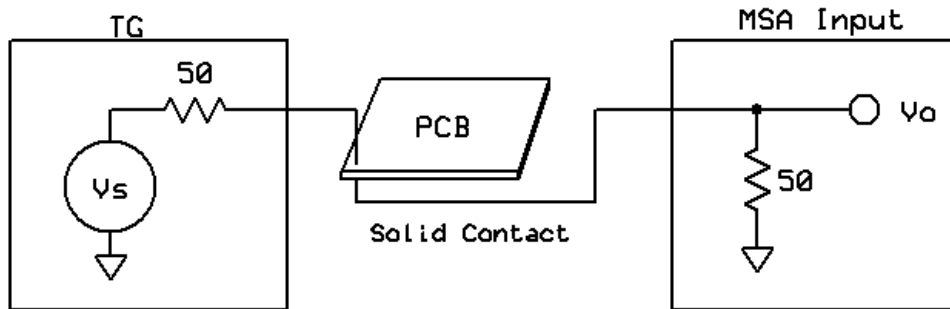
The Q drops significantly to 63, because the probes now load the resonance more heavily. The peak frequency dropped a bit, but not to have much affect on calculated ϵ , and certainly not as much as Eq. 6 might indicate. This demonstrates that the Q referred to in Eq. 6 is unloaded Q. It is the internal PCB losses that affect the signal propagation and the resonant frequency. Therefore, it is feasible to use the corner-to-corner method with direct coupling to the PCB planes. However, the peaks become more rounded, which makes them more difficult to identify where they are closely spaced.

Appendix B—Schematic of Test Methods

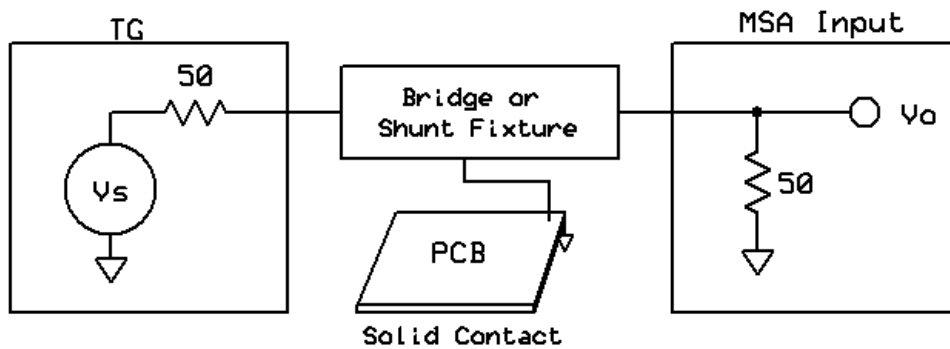
Corner-to-Corner Transmission



Plane-to-Plane Transmission



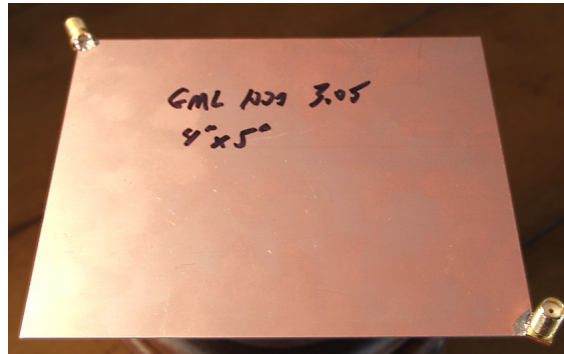
Reflection



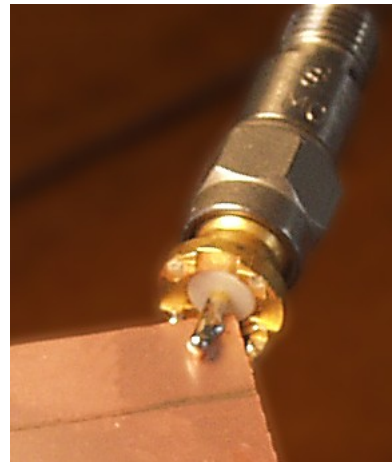
(TG is the Tracking Generator output.)

APPENDIX C—CONNECTION TO PCB

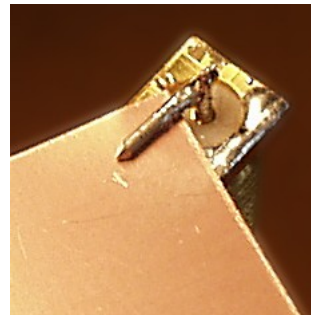
The following illustrates several different methods to couple the signal to the PCB. SMA connectors are placed in the corners so as to attach the body to one side and couple the center pin to the other side, without the center pin actually touching the PCB.



The upper left connector is mounted edgewise. This works well for thin PCB, which allows space between the PCB and the pin. If the space is too great, a blob of solder on the center pin will thicken it and reduce the space. The spacing is not critical; resonance peaks from -70 dB to -10 dB are workable. An optional 3 dB attenuator is shown in this photo. For reflection measurements of thick PCB, the SMA connector will contact both sides, as desired.



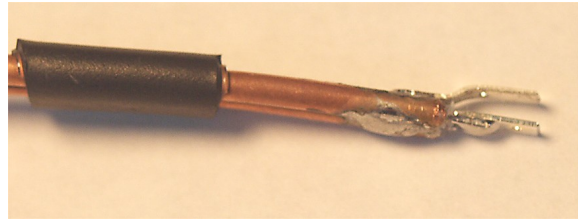
For thicker PCB, such as ordinary 0.062" FR-4, the connector can be mounted vertically as shown in the lower right of the above picture. A wire is soldered to the center pin so as to project over the PCB. The wire is not touching the PCB.



A solder-free method uses a “clamp” made by soldering rigid metal strips to a coax. The PCB is placed between them. A thin insulator, such as plumber’s Teflon tape, can prevent direct contact on one side. But if reflection is measured, contact on both sides is desirable.



Two semi-rigid cables with their shields soldered together and silver strips soldered to the shields and center conductors (so the center conductors are shorted together) makes a good Shunt Fixture that can be used for reflection measurements. A wire or piece of metal of appropriate thickness makes a good calibration short. A calibration load can be made by soldering two 0805 or 0603 resistors sideways on the edge of a small piece of PCB.



The same concept can be used for a Series Fixture, leaving the center conductors exposed. The PCB is placed between the leads and twisted to make good contact. This is perfect for the plane-to-plane transmission method. To make the through connection for calibration, just press both pins against one side of the PCB to short them together.

