Introduction to the Smith Chart for the MSA

Sam Wetterlin

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Quick Review of Reflection Coefficient

The Smith chart is a method of graphing reflection coefficients and impedance, and is often useful to examine the relationship between them. As a quick review, here are some formulas. First, the reflection coefficient Γ (gamma) is a complex number calculated from impedance by this formula:

$$\Gamma = \frac{Z - 50}{Z + 50} \qquad \text{(Equation 1)}$$

The "50" in the equation is the "reference impedance", which can have other values, but we will stick to 50 in this document.

Equation 1 requires the impedance to be in rectangular format, and produces Γ in that format. But G is almost always converted to polar format, with magnitude ρ (rho) and angle Θ (theta):

$\Gamma = \rho \angle \Theta$ (also written ρ @ Θ degrees, or rho at theta degrees)

Rho takes on values from 0 to 1, and theta can be any angle.

For completeness, here is the inverse of Equation 1, for converting reflection coefficient to impedance.

$$Z = 50 \times \frac{1 + \Gamma}{1 - \Gamma} \qquad \text{(Equation 2)}$$

Again, the "50" is really the reference impedance Z_0, which we assume here to be 50 ohms.

Basics of the Smith Chart

In the days of slide rules, the Smith chart was packed with lines, curves, grids and nomographs. The MSA uses a much simpler Smith chart. The basic chart is shown in Figure 1.

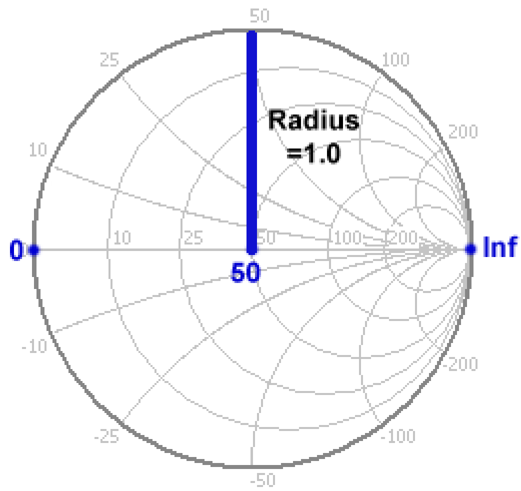

Figure 1—The Smith Chart

The Smith chart is a circle whose radius is considered to be one. The center of the chart is the origin. Reflection coefficients run from 0 to one, with an angle from -180 to 180 degrees. This makes the Smith chart perfect for graphing reflection coefficient. To graph a value rho @ theta degrees, we move a distance rho from the origin in the direction indicated by theta. Theta is measured from the axis running to the right of the chart, with positive angles moving counter-clockwise. Thus, 50 ohms is located at the center, because its reflection coefficient is zero. An infinite resistance has a reflection coefficient of 1 @ 0 degrees, which puts it on the boundary to the right of the origin. A resistance of zero also has a reflection coefficient of 1, but an angle of 180 degrees, which puts it on the boundary to the left of the origin.

The points 0, 50 and infinity are always good to keep in mind when thinking of the Smith chart.

It may seem odd that we are graphing reflection coefficient but there are no grid marks to indicate what angle we are at or how far we are from the origin. A full Smith chart would have such markings. But since the MSA draws the graphs for us, we don't need the markings. It is more useful to have the chart marked with impedances. Figure 1 shows resistance values marked along the horizontal axis, and reactance values marked along the boundary.

<u>Constant Resistance Circles and Constant Reactance Arcs</u>

To interpret impedance on the Smith chart, it is necessary to understand constant resistance circles and constant reactance arcs. Figure 2 demonstrates the concept.

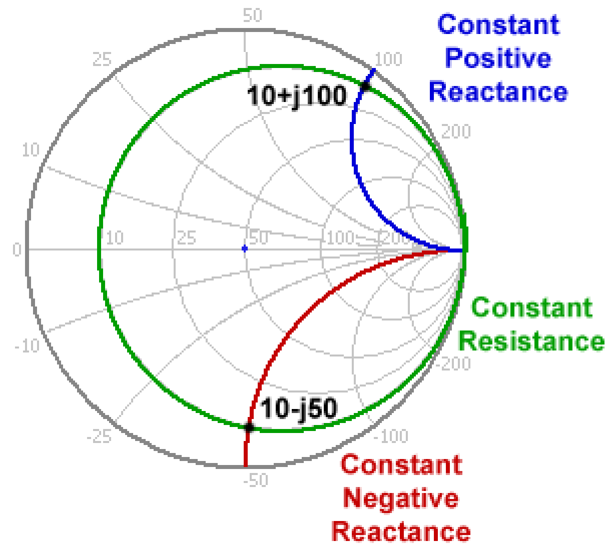

Figure 2—Constant value circles and arcs

Focus first on the green circle of Figure 2. It is a constant resistance circle of value 10, as marked on the horizontal axis. Every possible impedance of the form 10+jX, where X is a reactance, is located on that circle. There are many other such circles drawn on the chart; the values become bigger, and the circles smaller, as you move to the right, and the large value circles collapse toward the infinity point. In fact, even the boundary of the chart is a constant resistance circle, of value 0. Values such as 12.5 ohms that are not shown on the chart also have such a circle. Using the drawn circles you can get a rough idea where they are located.

Now look at the blue arc, which is a constant reactance circle of value j100, as marked at the boundary (the "j" is not shown, but all values on the boundary have an implied "j"). Every possible impedance of the form R+j100 (where R is a resistance) is on that little arc. The fact that the imaginary component is positive indicates that the impedance is inductive. There are many other such arcs drawn on the chart. The horizontal axis itself is a constant reactance arc of value zero; it's an arc of infinite radius. As with resistances, the smaller inductance values have large circles; as the reactance increases the arc radius becomes smaller and the arcs collapse toward the infinity point.

The red arc is a constant reactance arc of value -j50. All impedances of the form R-j50 are located on that arc. It has the same nature as the blue arc, but because its value is negative, it indicates a capacitive impedance.

To read the value of impedance, you determine what constant resistance circle intersects with what constant reactance arc at that point. For example, Figure 2 shows that the intersection of the green and red lines is 10-50j.

In effect, the horizontal center line is the resistance axis, and the circular boundary is the reactance axis. Graphing impedance on the Smith chart is exactly like graphing on a normal X-Y graph, except the resistance axis is compressed and the reactance axis is curved.

Figure 3 shows two more reflection coefficients graphed on the Smith chart, to illustrate how to relate reflection coefficients to impedances using constant resistance circles and constant reactance arcs.

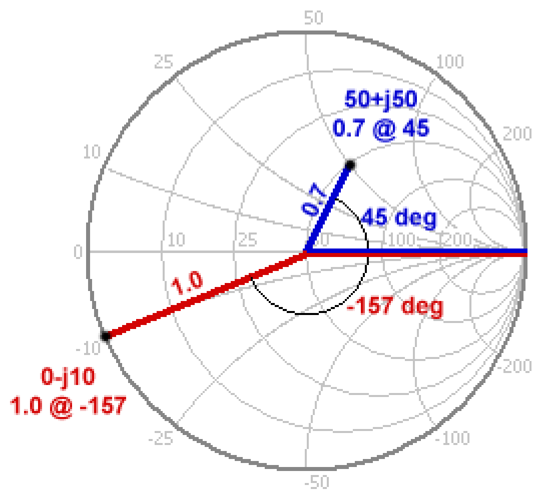

Figure 3—Two more points graphed on the Smith chart

The blue lines show the graphing of the reflection coefficient 0.7 @ 45 degrees. The point is located 0.7 units from the origin, at 45 degrees. That point lies on the constant resistance circle for 50 ohms, and the constant reactance ohm for j50 ohms. Hence its impedance is 50 + j50 ohms. Similarly, the red lines show a reflection coefficient of 1 @ -157 degrees, putting us on the boundary (constant resistance zero) and on the constant reactance arc for –j10 ohms. So the impedance is 0-j10 ohms, a pure capacitance.

Division of Smith Chart into Areas

Figure 4 shows the principal areas of the Smith chart and the type of impedance they represent.

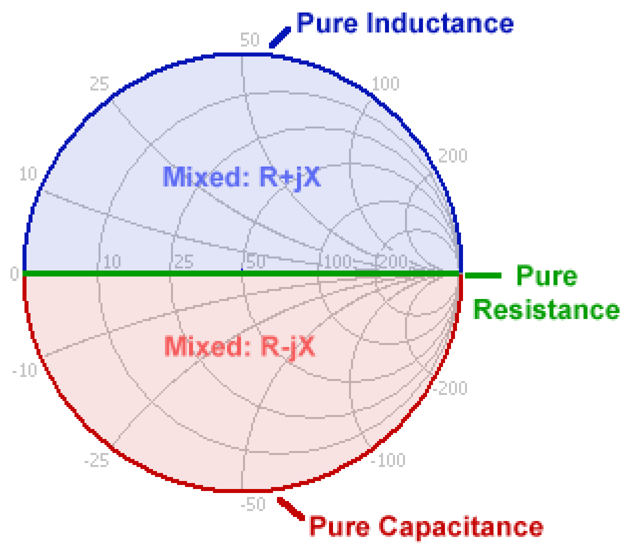

Figure 4—Principal Areas of the Smith Chart

We have already seen that the horizontal axis is a constant reactance circle of value 0. That makes all impedances on the axis pure resistances, as shown by the green line in Figure 3. All the positive constant reactance arcs are in the upper half of the chart, making all those impedances inductive. All the negative constant reactance arcs are in the lower half, making all those impedances capacitive. Pure capacitances and inductances have a reflection coefficient of 1, lying at various angles. That means all inductors graph to the upper dark blue boundary, and all capacitors graph to the lower dark red boundary.

Constant Rho Circles

There is another very important set of constant-valued circles. Because we are graphing reflection coefficients by moving a distance rho from the center at the appropriate angle, by definition all reflection coefficients with the same rho value will be the same distance from the center, and hence constitute a circle. Figure 5 shows the constant rho circle of value 0.33.

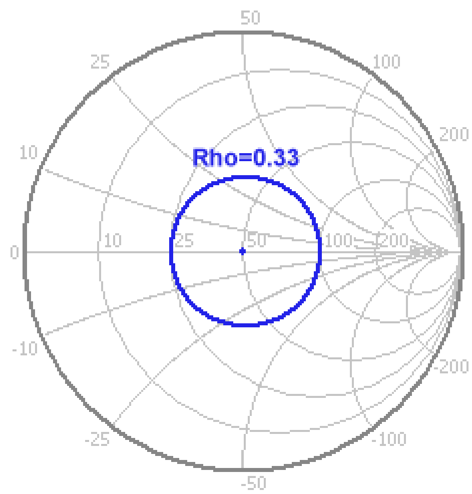

Figure 5—A constant rho circle

Note that the circle for rho=0.33 includes pure resistances 25 and 100. The circle will always include two pure resistances (except for rho=0, which produces a dot). If the lower value is 50/N, the upper value is 50*N.

Effect of Transmission Line on Impedance--Theory

Consider what happens when you measure the input impedance of a 50-ohm coax, terminated by a 100-ohm resistor. The resistor will reflect 33% of the incident signal voltage at zero angle. With a very short coax, the reflection will still be at nearly zero angle when it returns back to MSA. The MSA will conclude that the reflection coefficient is 0.33 at zero degrees, and that the impedance is 100 ohms. If you lengthen the coax, the reflection at the resistor will be the same as before. However, the signal will have some phase delay before it reaches the resistor, and the reflection will have some more phase delay before it reaches the MSA. At the MSA, rho will have the same value as before, but theta will include twice the phase delay of the coax (meaning theta will be decreased; delay in phase is negative). The MSA will conclude that the reflection coefficient is now, say 0.33 at 90 degrees. This puts the impedance at the bottom of the blue circle in Figure 5. If the coax is exactly a quarter-wavelength, there will be a 90 degree delay for each travel through the coax, so the reflection will be delayed 180 degrees, putting rho right on top of 25 ohms. This demonstrates the familiar fact that a quarter-wave transmission line transforms the terminating impedance from 50*N to 50/N.

We could also do this experiment with an open transmission line, which would show infinite impedance for a very short line, putting it at the infinity point at the right of the Smith chart, on the boundary (which is a constant rho circle of value 1). The same 180 degree rotation caused by the quarter wavelength line would move the graphed reflection coefficient around the bottom of the chart to the zero point on the far left. This confirms the transformation by a quarter-length line of an open into a short, and vice-versa.

The fact that 50-ohm transmission lines simply rotate the graphed reflection coefficient around a constant-rho circle makes the Smith chart a very useful tool for analyzing the effect of transmission lines. The precise effect can be calculated by a computer, but the Smith chart is a great aid in understanding the concept.

Effect of Transmission Line on Impedance--Actual

In the thought experiment we just conducted, we imagined increasing the length of the transmission line to cause greater phase delay. We can achieve the same effect by increasing the frequency, since shorter wavelengths convert to greater phase delays for any given line length.

So much for thought experiments. Let's try the real thing. Figure 6 is a graph of a coax cable terminated with 25 ohms.

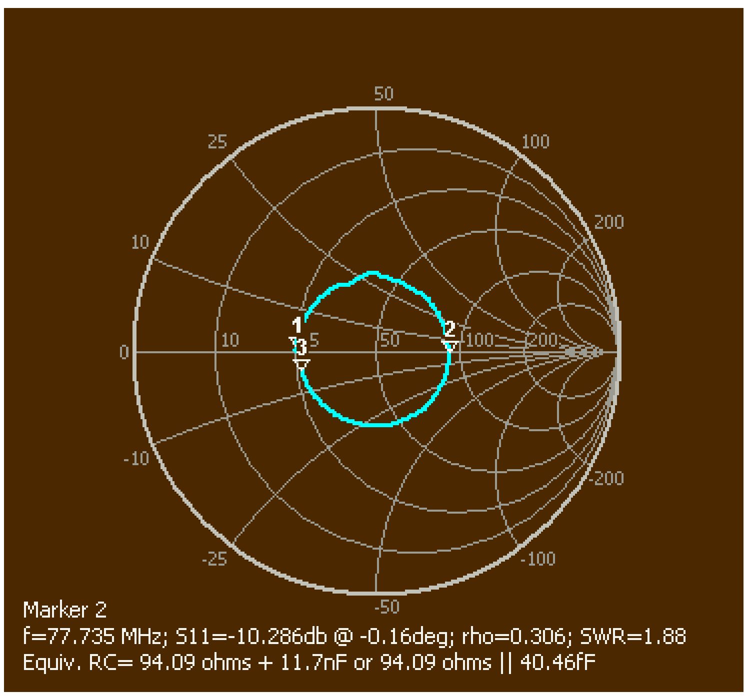

Figure 6—Reflection of coax terminated with 25 ohms

Low frequency (1 MHs) is marker 1; high (150 MHz) is marker 3

The graph in Figure 6 starts at 25 ohms (marker 1), moves to marker 2 when the quarter-wave frequency is reached, and continues on to marker 3. (In almost all Smith chart graphs—maybe literally all of them—the motion as frequency increases, if there is any motion, is clockwise.) Note that the resistance at marker 2, displayed on the bottom, is only 94 ohms, so it did not reach the theoretical 94 ohms, due to coax losses. Still this illustrates the rotation effect of coax cable.

Figure 7 shows a similar graph, using an unterminated coax.

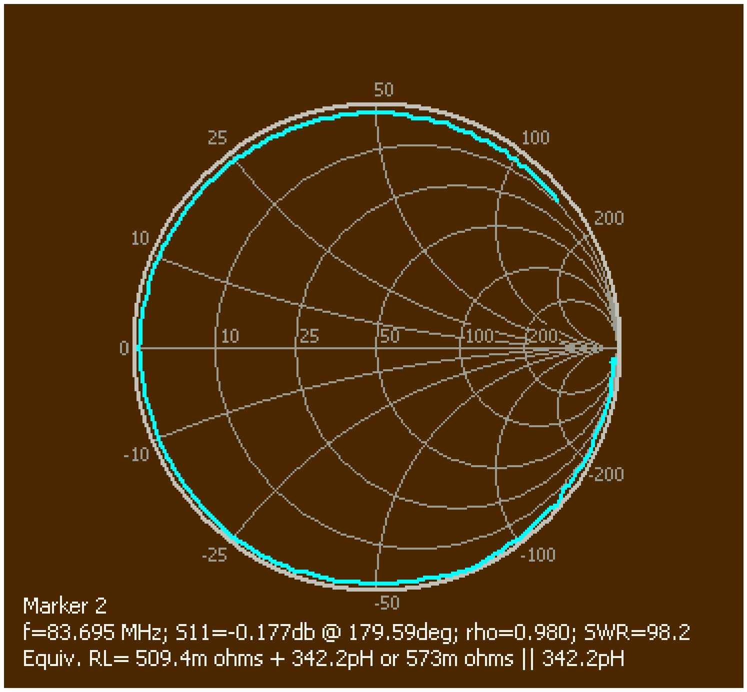

Figure 7—Unterminated coax, 1-150 MHz

The markers got turned off on Figure 7, illustrating one weakness of Smith chart graphs: without markers, you have no idea what frequency is at what point, except you know the action moves clockwise as frequency increases. It starts near the infinity point—not exactly there, because we started at 1 MHz. It rotates clockwise, gradually falling a little further from the boundary due to loss. So it comes close to following the constant rho circle of value 1. Marker 2 was actually located near the zero point, where the graph caused the resistance axis. The displayed resistance is 0.5 ohms, plus a trivial amount of inductance.

Finally, Figure 8 shows a similar graph with the coax terminated with 60 ohms.

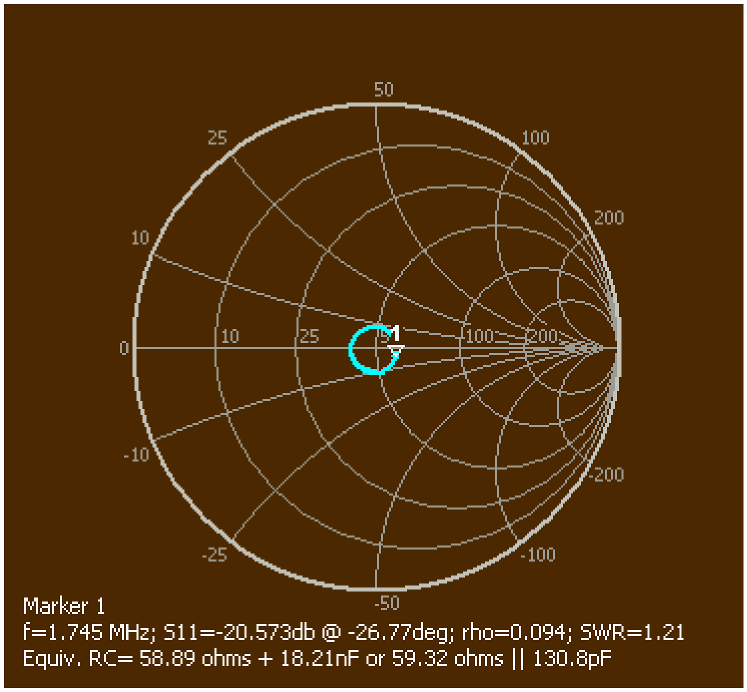

Figure 8—Coax terminated with 60 ohms.

Figure 8 shows a nice circle around the origin caused by the rotating effect of the coax cable. Generally, the closer the terminating resistance is to 50 ohms (or whatever the actual characteristic impedance of the coax is), the cleaner the circle will be.

Graphs Following Constant Resistance Paths

Graphs of coax cable are not the only ones that tend to follow a constant rho path on the Smith chart. Figure 9 shows a 100 pF capacitor

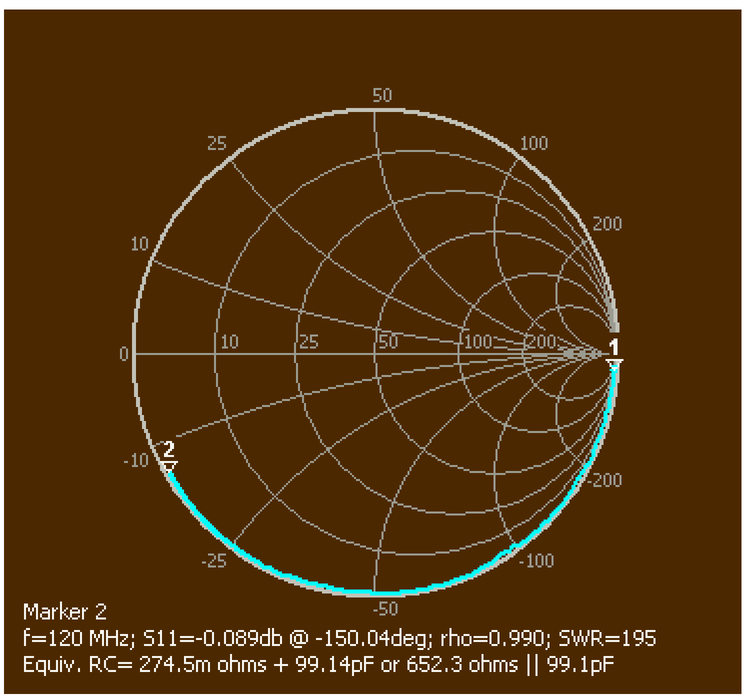

Figure 9—100 pF capacitor, 1-150 MHz

Starts at marker 1

The capacitor in Figure 9 starts on the boundary near the infinity point and moves along the boundary (which is both a constant resistance (0) circle and a constant rho (1) circle). As its impedance decreases, it moves toward the zero point.

Figure 10 shows a ferrite core inductor of unknown value (but apparently it is near 300 nH).

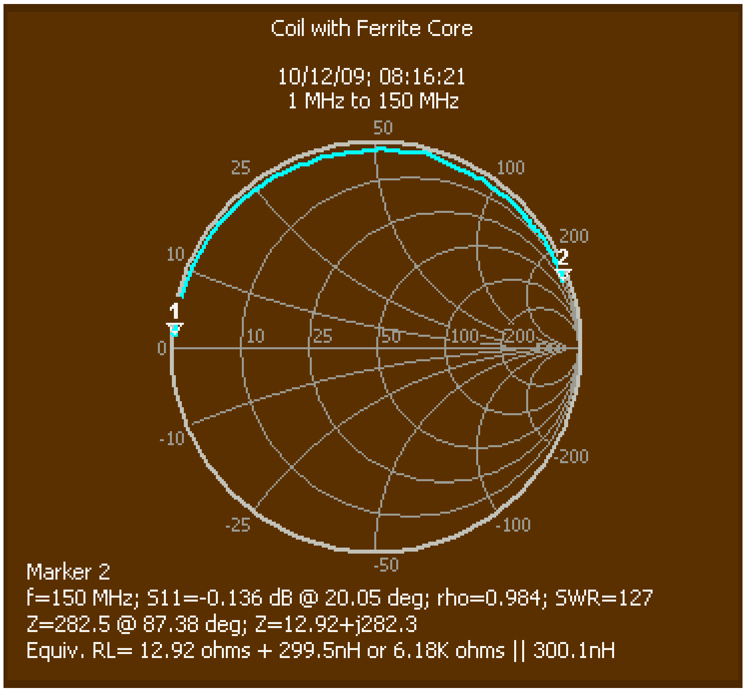

Figure 10—Unknown Coil
This later software version shows additional information

The coil starts at 1 MHz with low impedance near the zero point, then approximately follows the top of the chart to point 2. The displayed values show that the coil has a value of 300 nH and series resistance of 13 ohms. The resistance is the reason it does not follow the top boundary perfectly.

Figure 11 shows that same coil with an added 18 ohm series resistor.

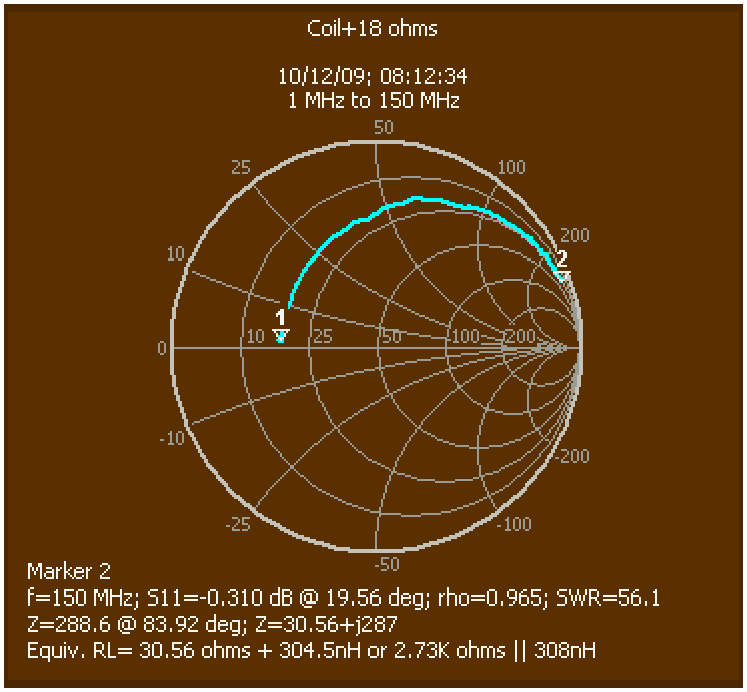

Figure 11—Coil with series 18-ohm resistor

The graph of Figure 11 should ideally start near 18 ohms and follow the 18-ohm constant resistance circle. However, the ferrite losses, which increase with frequency and were measured in Figure 10 at 150 MHz as 13 ohms, cause some deviation. Note that the measured series resistance in Figure 11 is 30.6 ohms, very close to the coil's 13 ohms (measured in Figure 10) plus the resistor's 18 ohms (which actually has 5% tolerance).

Figure 12 shows the coil in parallel with an unmarked capacitor.

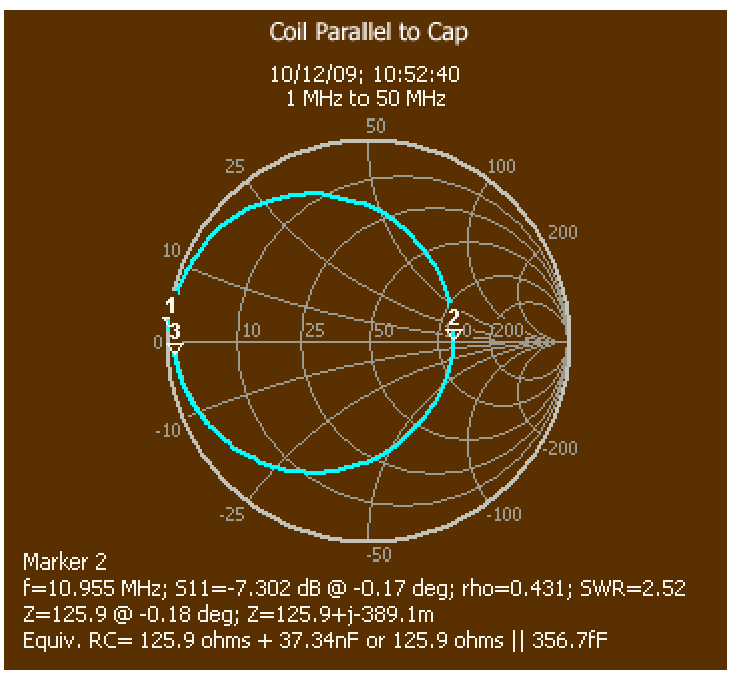

Figure 12—Coil plus capacitor

The LC combination in Figure 12 starts out looking like a small inductive impedance at 1 MHz and moves clockwise until it reaches marker 3 at 30 MHz. With no loss, the path would follow the Smith chart boundary. In between, at marker 2, the path crosses the resistance axis at 11 MHz; because this point has no net reactance, it is the point of resonance. The resistance at that point is 126 ohms; far from the ideal of infinity for parallel resonance; this is due to the significant resistance of the coil. On a regular graph of transmission, this device would show a broad modest peak or dip at resonance (depending how it is hooked up). But with the Smith chart, the resonance shows up as a dramatic circular movement, as the parallel LC combination moves from being a net inductance to being a net capacitance. The point of resonance is very obvious on the Smith chart.

A Word on the Infinity Point

We have been referring to the rightmost point on the resistance axis as the infinity point, because resistances get larger and larger as you approach that point on the resistance axis. Reactance arcs also collapse toward that point as the reactance gets huge. However, that point is actually somewhat of a twilight zone, because every constant resistance circle and constant reactance arc also terminates at that point! Every impedance value that has a huge resistive or reactive component is located very near the infinity point. It's not that we are worried that we might actually have an impedance with an infinite component, but the infinity point illustrates a general problem with reflection coefficients. If you have a high impedance that is located close to the infinity point, a very tiny movement of that point can represent a huge change in impedance. For instance, the distance on the Smith chart between 10K ohms and j*10K ohms is miniscule. All high impedances are crammed into the right "corner" of the Smith chart. When measuring reflection coefficient with a reflection bridge, the tiniest measurement error translates into a huge impedance error.

The bottom line is that high impedances are just not suitable for measurement by means of the reflection coefficient, which is most accurate for measurement of impedances in the general neighborhood of 50 ohms. As a practical matter, users of the Smith chart generally would consider anything near the infinity point as being just "bad", and they would make an effort to transform it closer to 50 ohms, and then measure how well they did.

Conclusion

We have introduced the basic concepts of the Smith chart used in the MSA, and shown how they can be interpreted by using constant resistance circles, constant reactance circles, and constant rho circles. Tests of actual devices show that their Smith chart graphs deviate from one of those constant-value curves by varying amount due to losses that make them non-ideal components. Nevertheless, graphs of devices with increasing frequency show a clockwise movement that is generally consistent with the ideal path.